

Battleship Complexity

GT GDMM 2021

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Joined work with Loïc Crombez and Guilherme Da Fonseca



Plan

I

Problem Statement

II

Shooting Algorithms

III

Results

IV

Proofs

Plan

I

Problem Statement

II

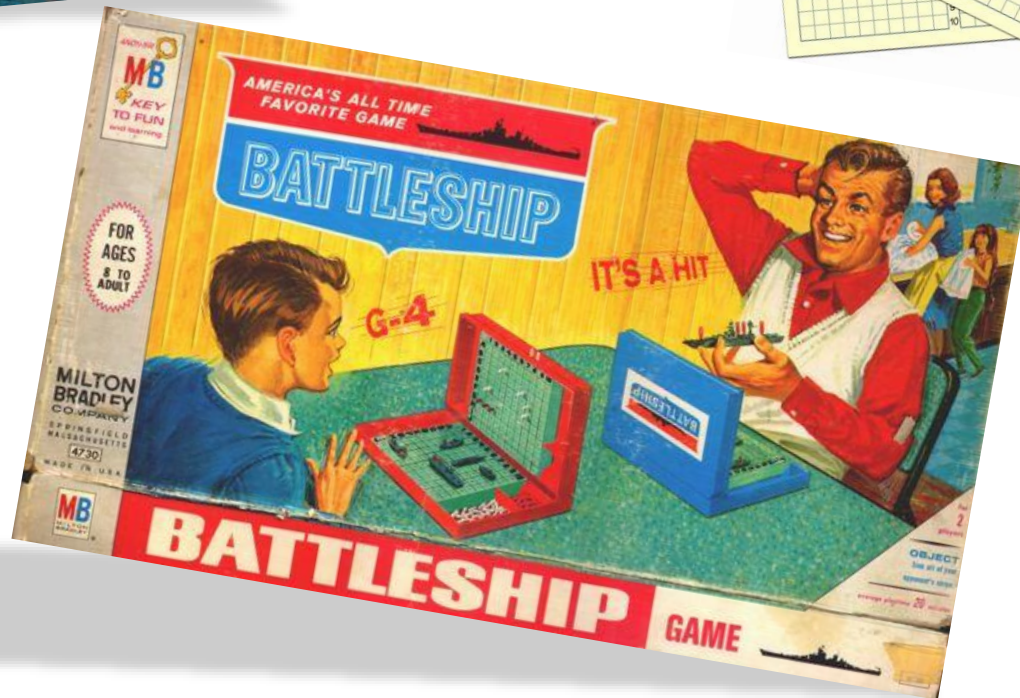
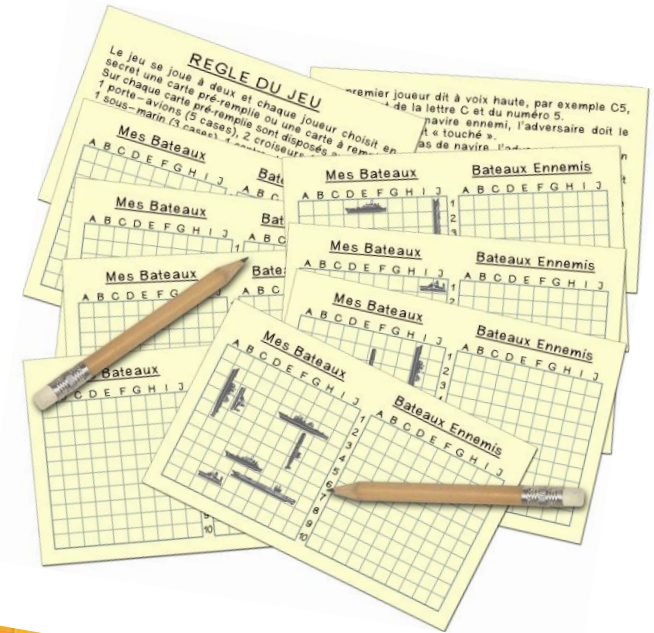
Shooting Algorithms

III

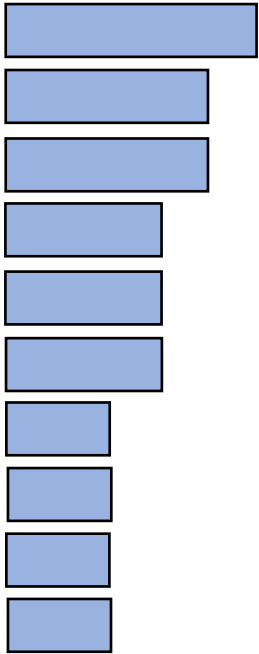
Results

IV

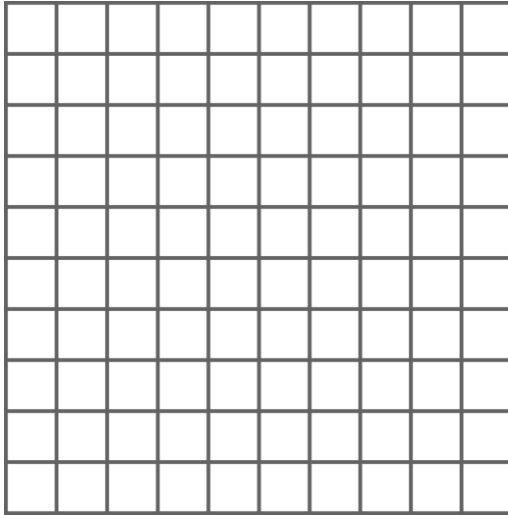
Proofs



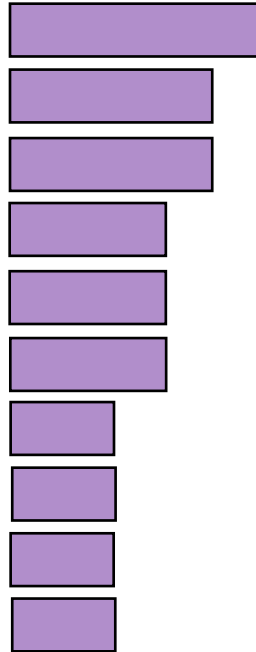
Your fleet



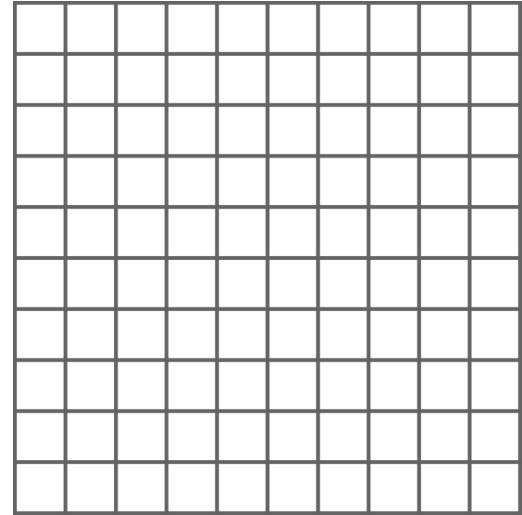
Your Battleships



Enemy fleet

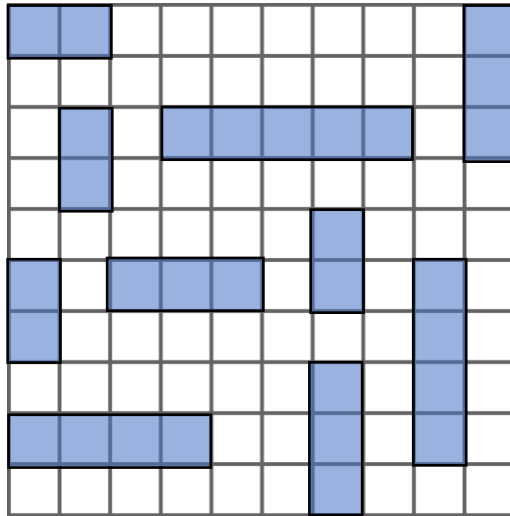


Enemy Battleships

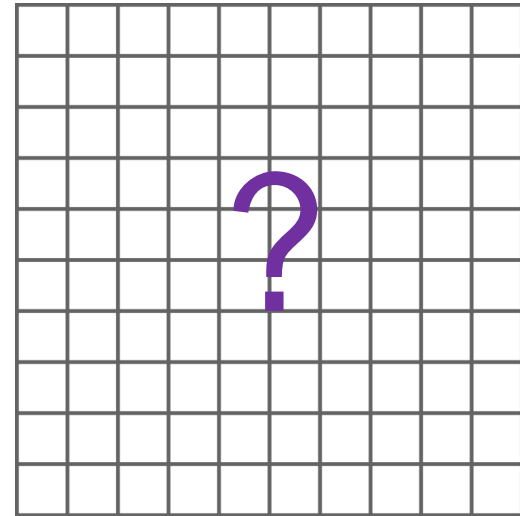


2 players : you and the enemy

Your Battleships

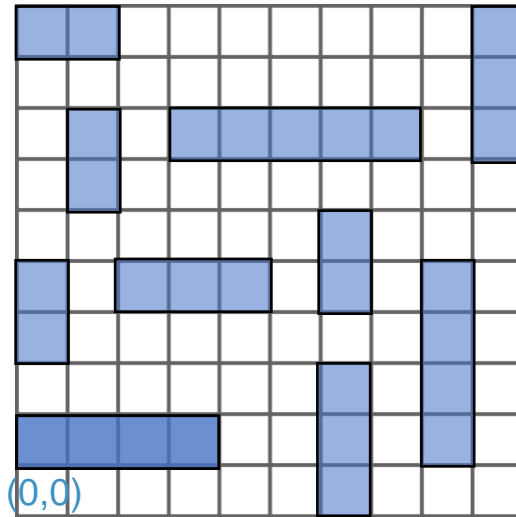


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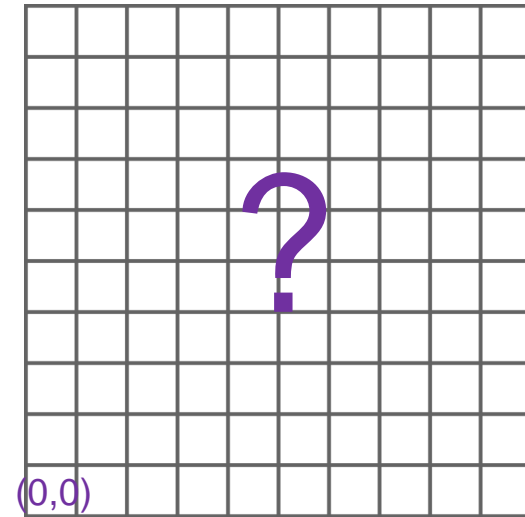


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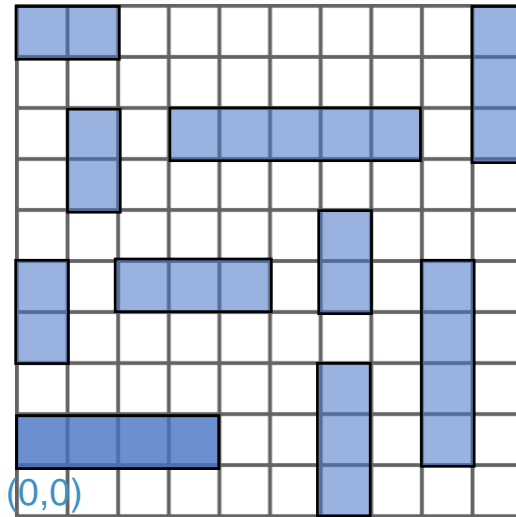
Enemy Battleships



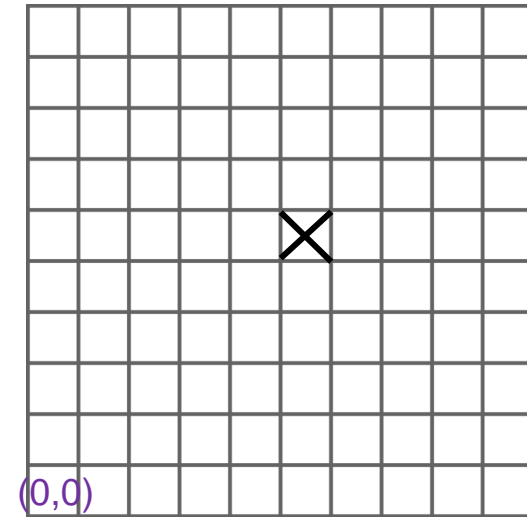
At each turn, **shoot** a square in your enemy grid => the enemy tells you if it is a **hit** or a **miss**

Your Battleships

Hit 
Miss 

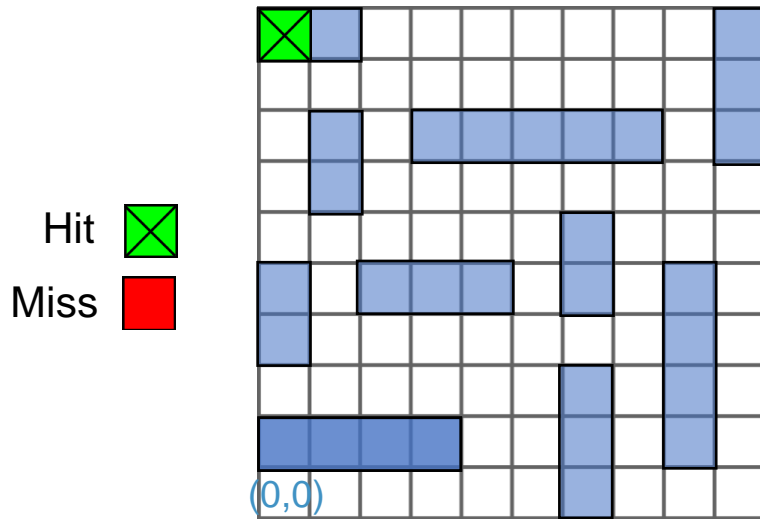


Enemy Battleships

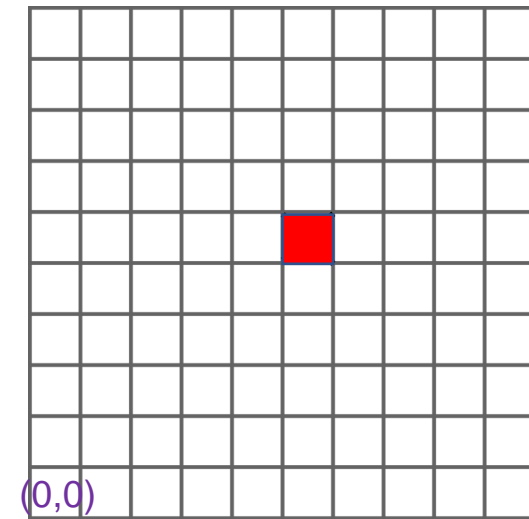


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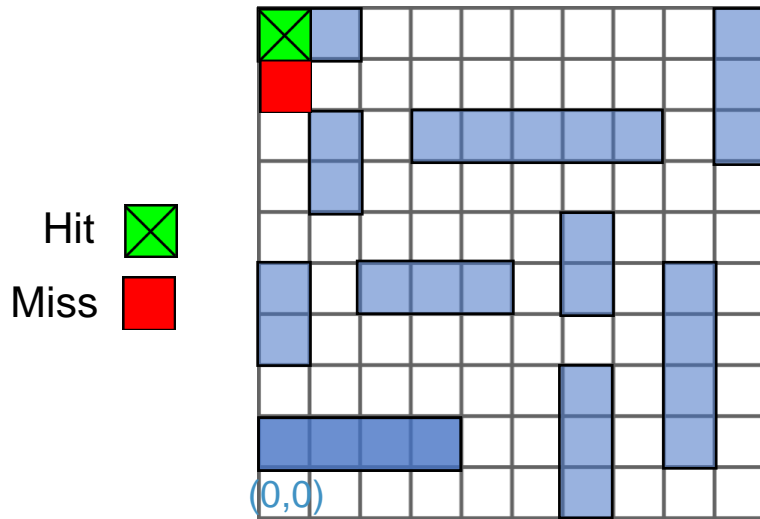


Enemy Battleships

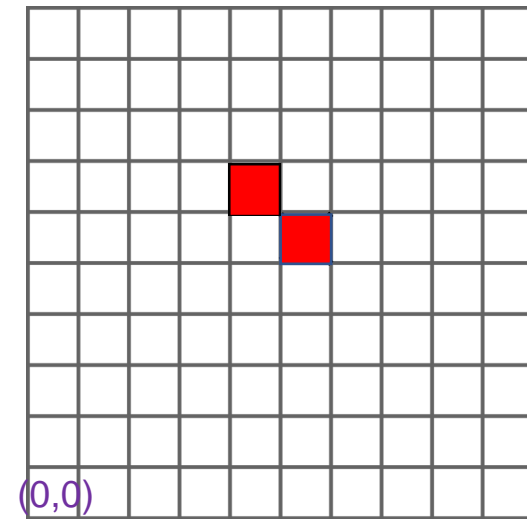


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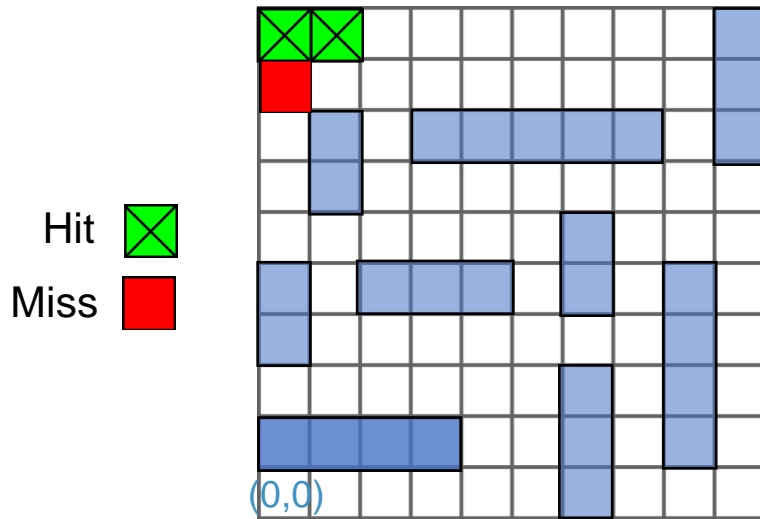


Enemy Battleships

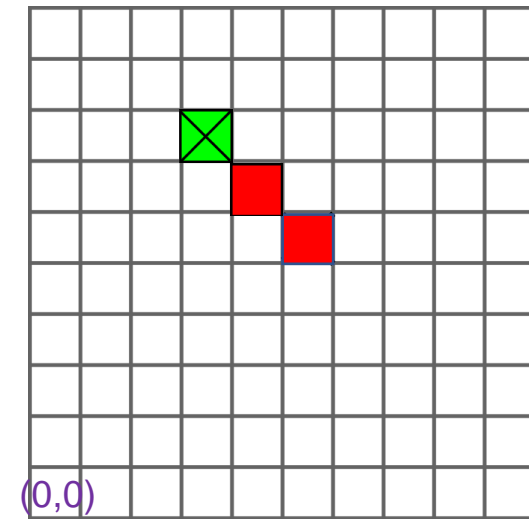


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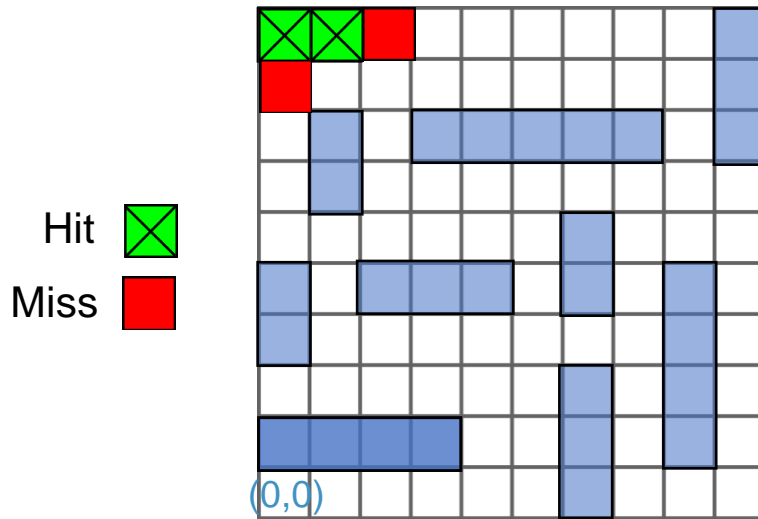


Enemy Battleships

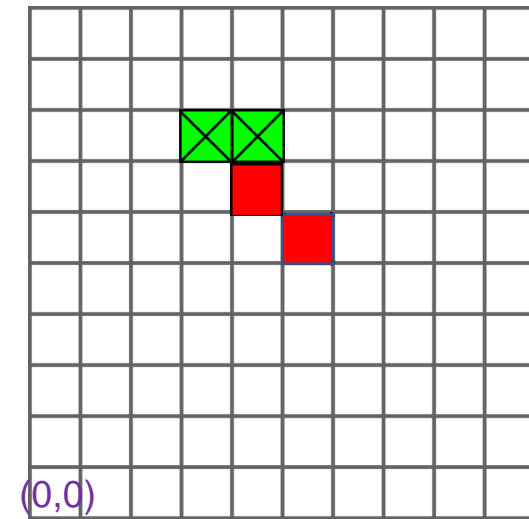


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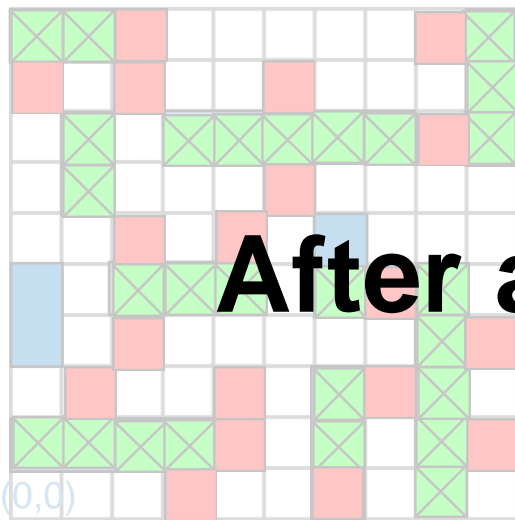


Enemy Battleships

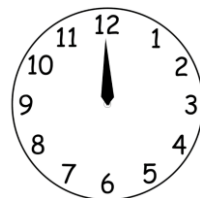


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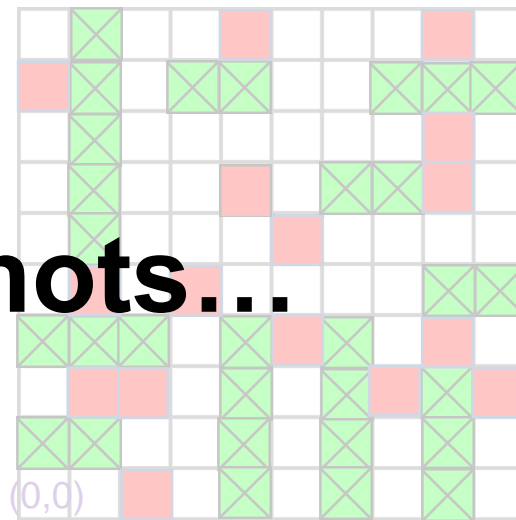
Your Battleships



Hit 
 Miss 



Enemy Battleships



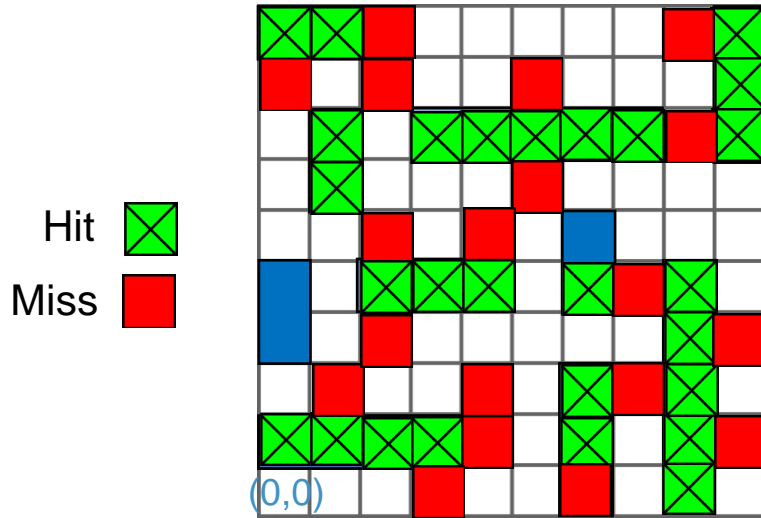
After a few shots...

You still have safe squares

All enemy ships have been sunk

At each turn, **shoot** a square in your enemy grid => the enemy tells you if it is a **hit** or a **miss**

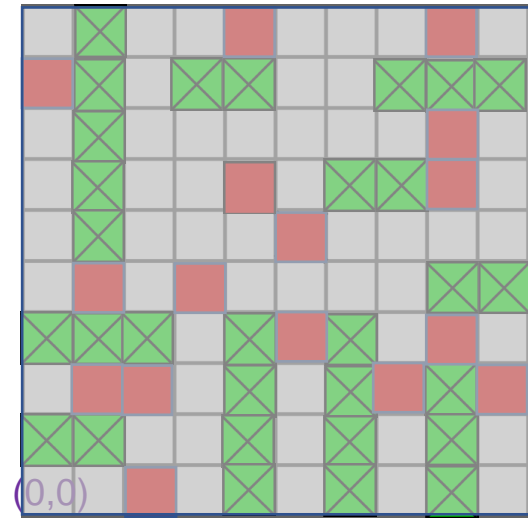
Your Battleships



You still have safe squares



Enemy Battleships




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Your Battleships

Enemy Battleships

Hit 

Miss 



You still have safe squares



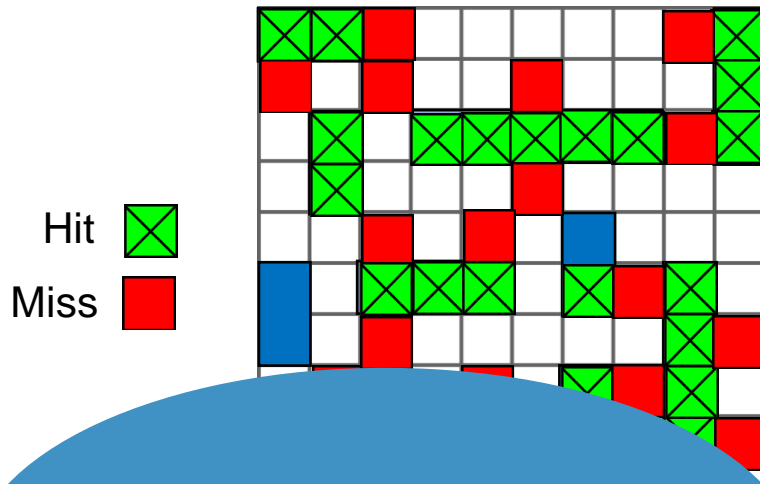
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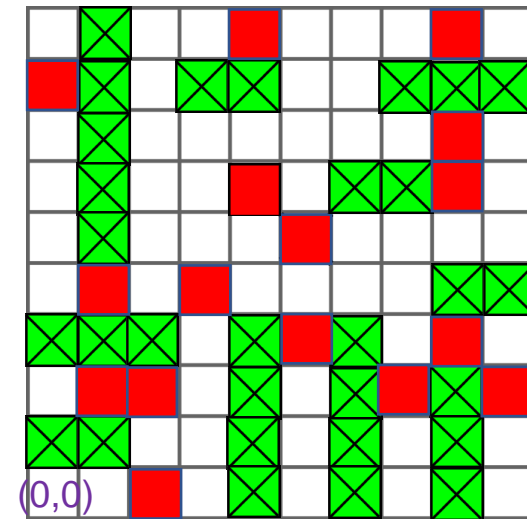
The winner is the first player which has hit all the squares of the enemy fleet

Your Battleships



We change the rules to focus on a specific algorithmic question

Enemy Battleships



All enemy ships have been sunk

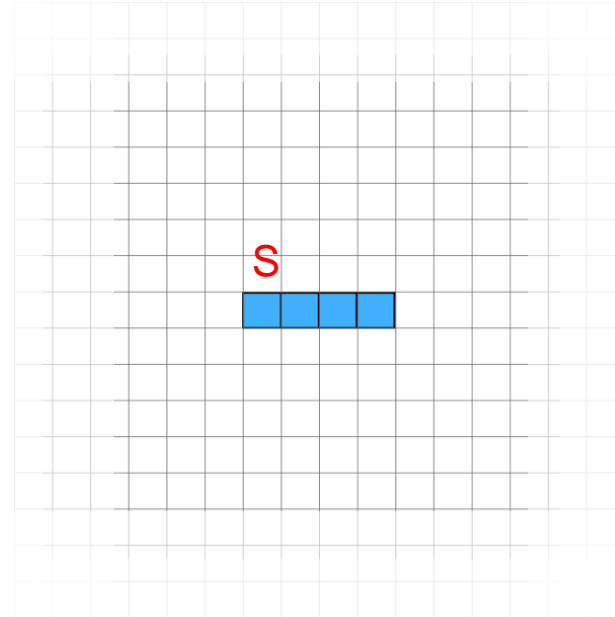


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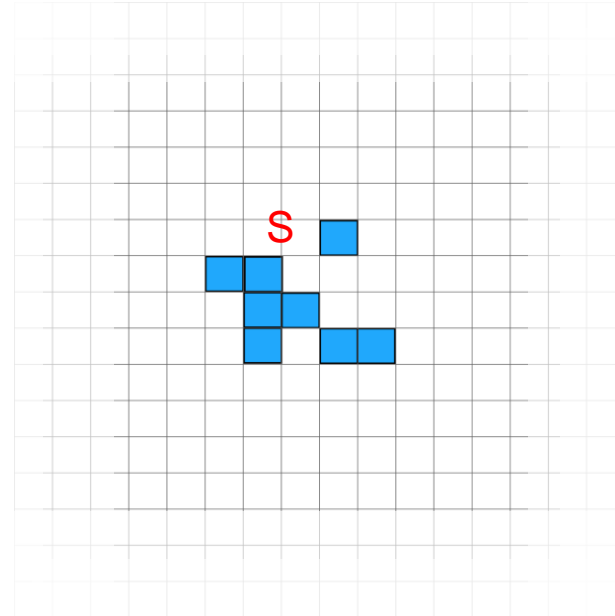
The **winner** is the first player which has hit all the squares of the enemy's fleet

- Only **one ship S** in an unbounded grid (no information comes from a position close to the boundary of the grid)

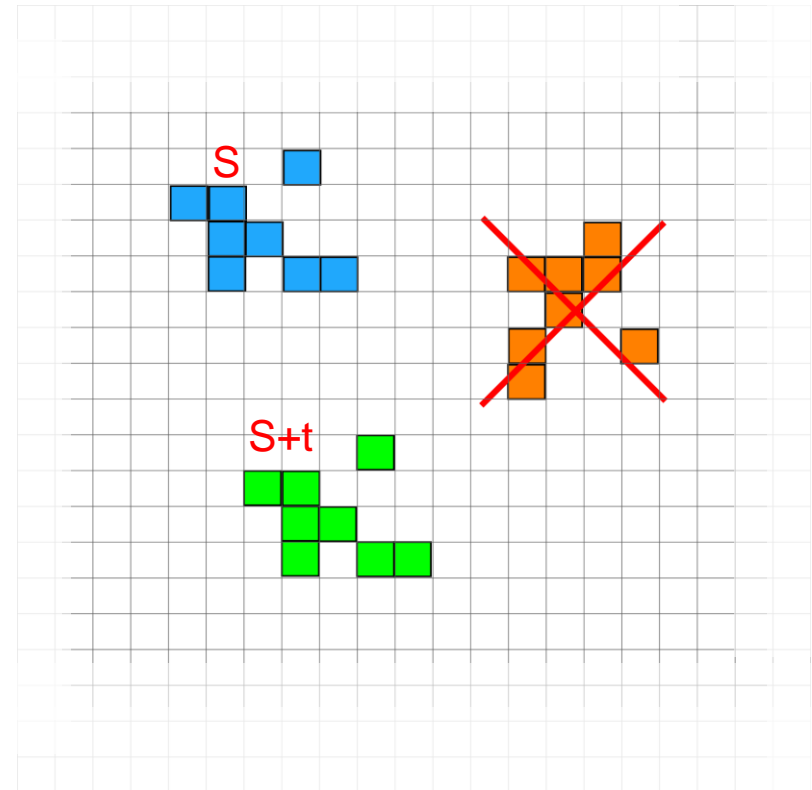
We change the rules to focus on a specific algorithmic question



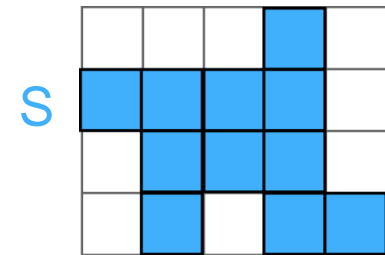
- Only **one ship S** in an unbounded grid (no information comes from a position close to the boundary of the grid)
- The shape of the ship is given but there is **no restriction** on it...



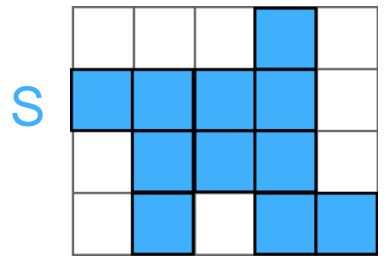
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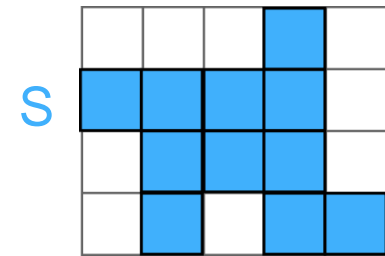
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Shape of the ship





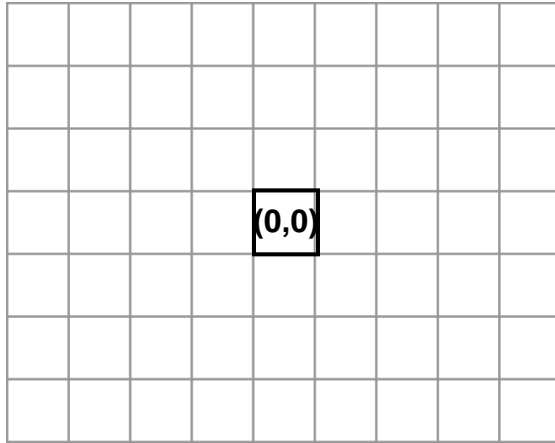
Shape of the ship



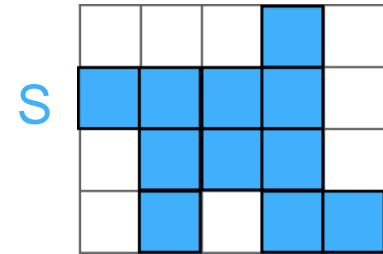
Shape of the ship

$$n = |S| = 11$$

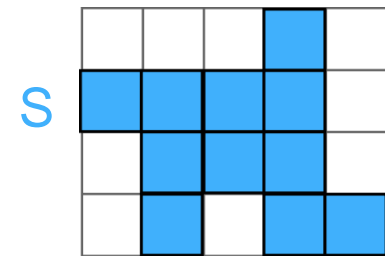
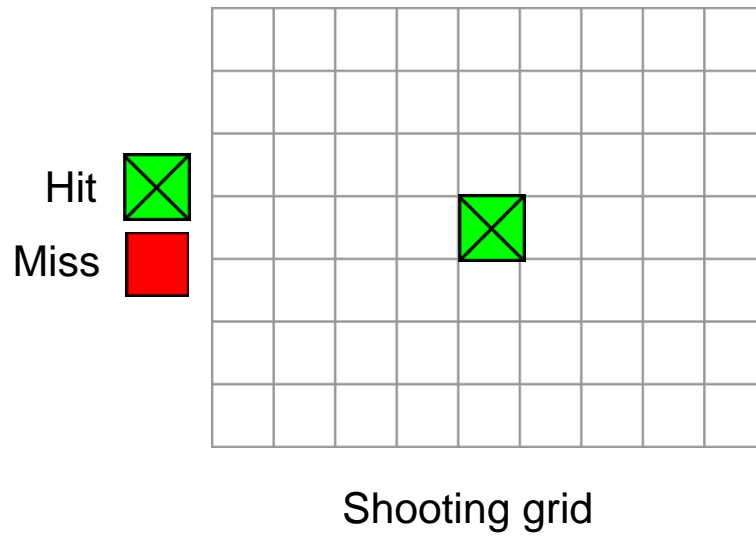
Hit 
Miss 



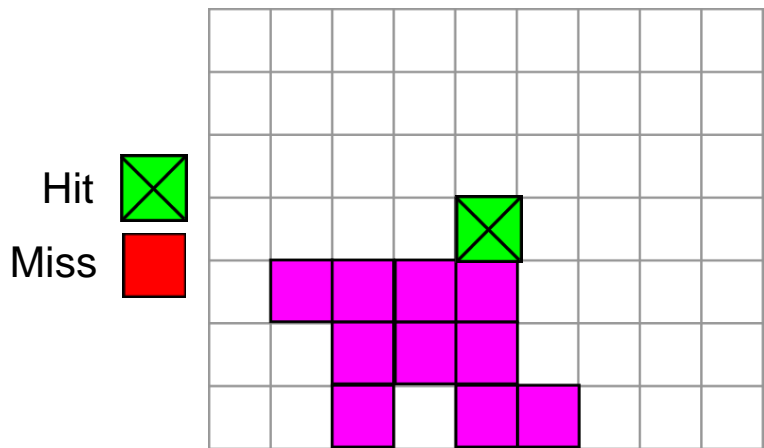
Shooting grid



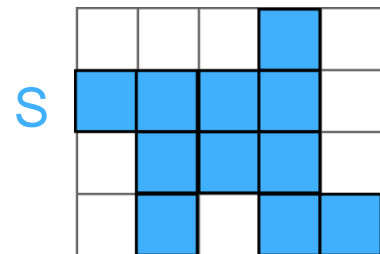
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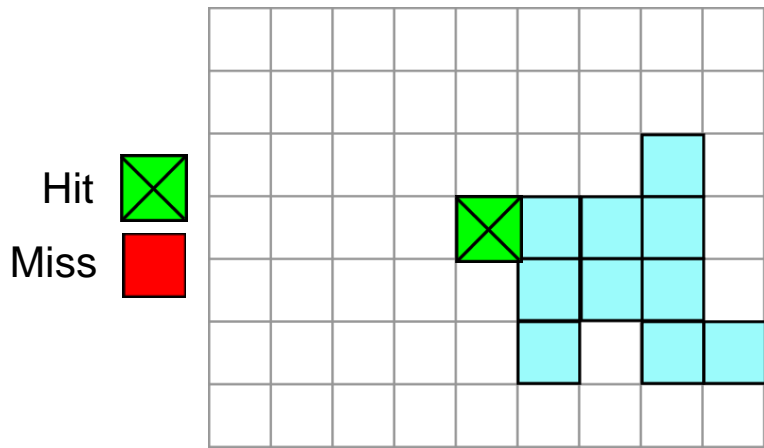
We assume that the ship has been **hit** one time by shooting at coordinates **(0,0)**



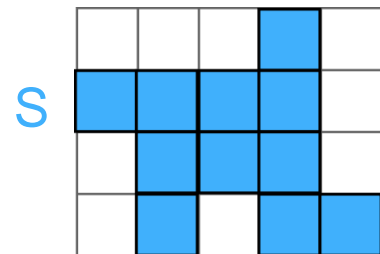
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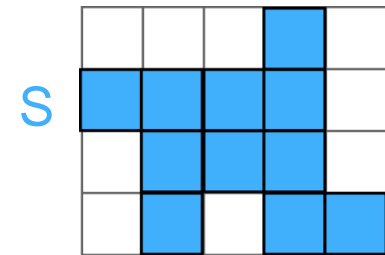
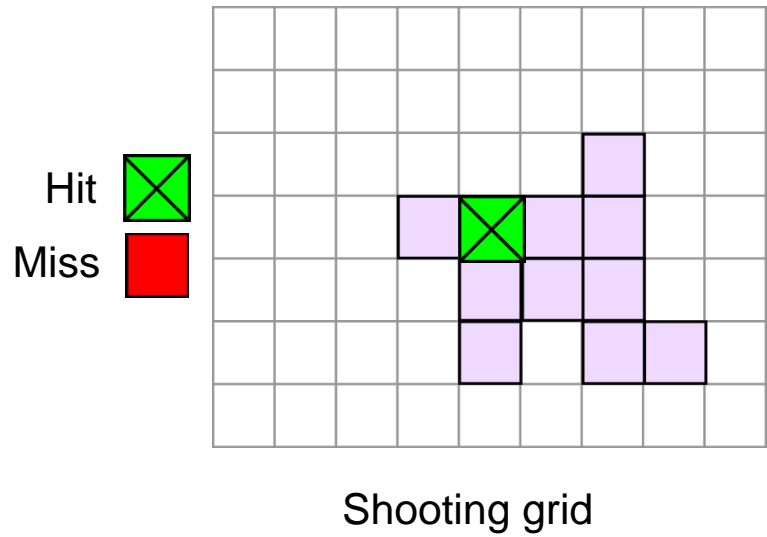
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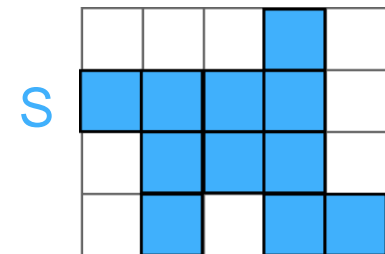
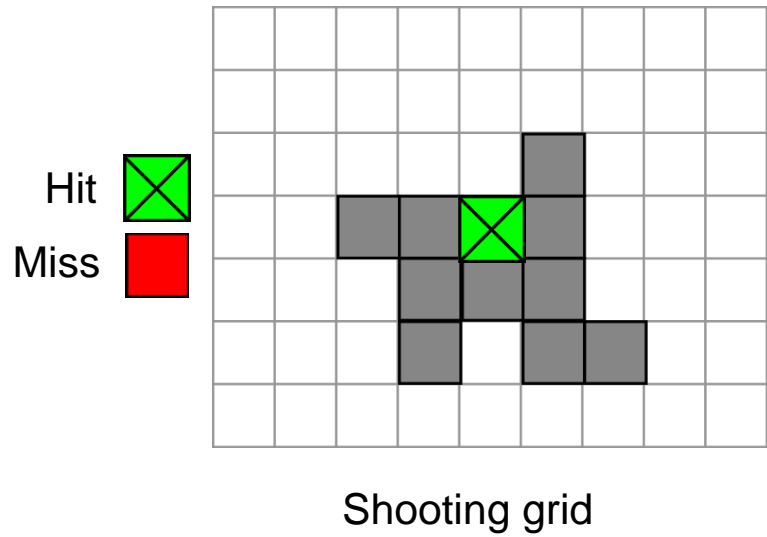
Shooting grid



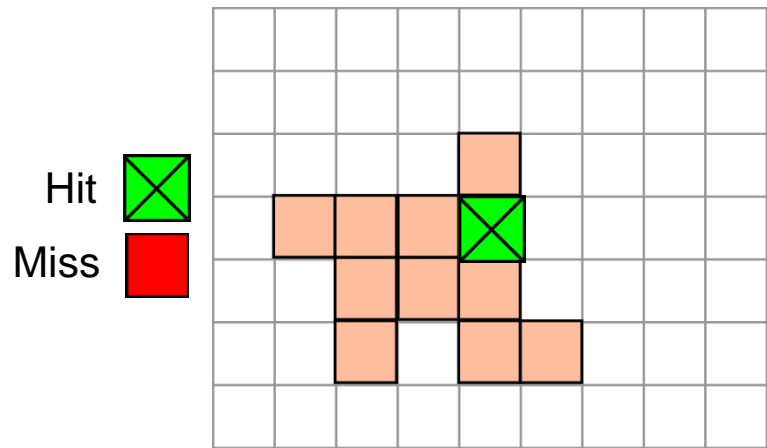
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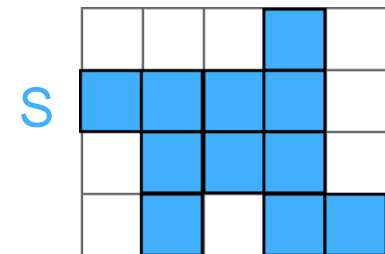
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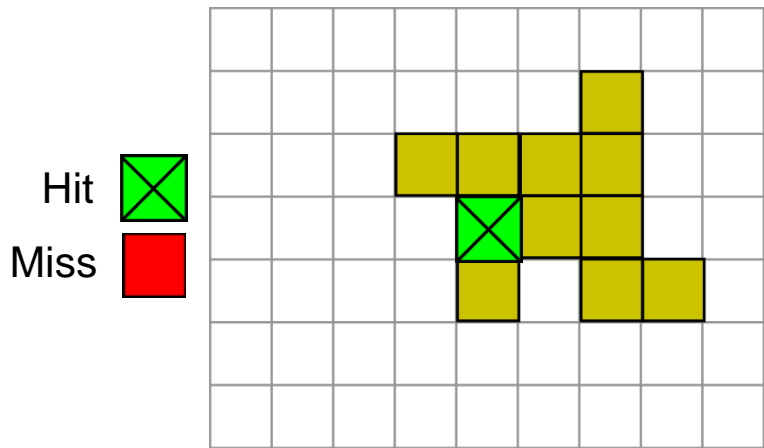
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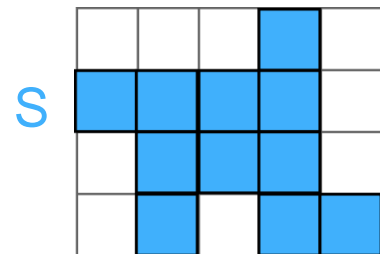
Shooting grid



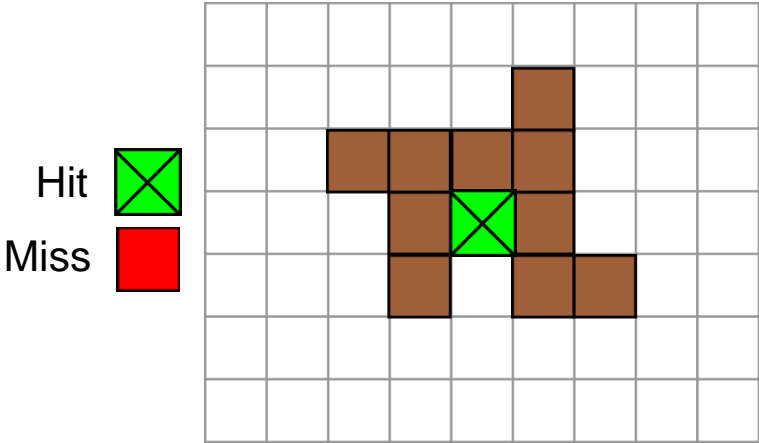
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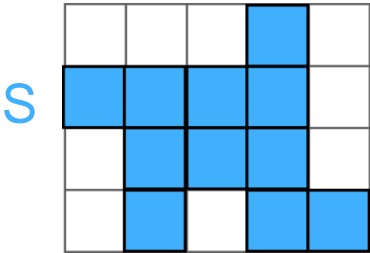
Shooting grid



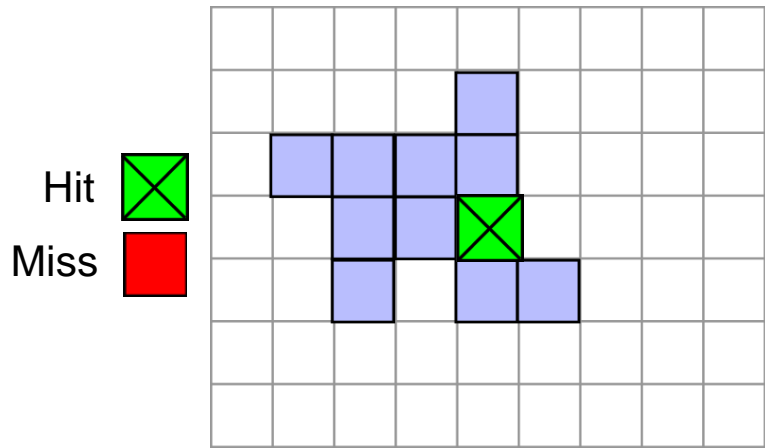
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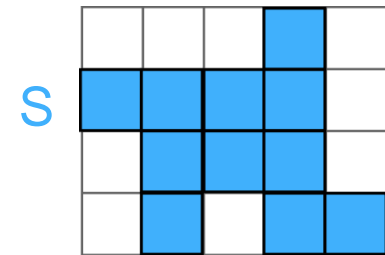
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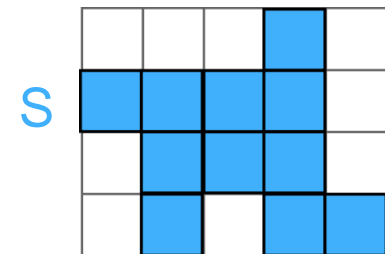
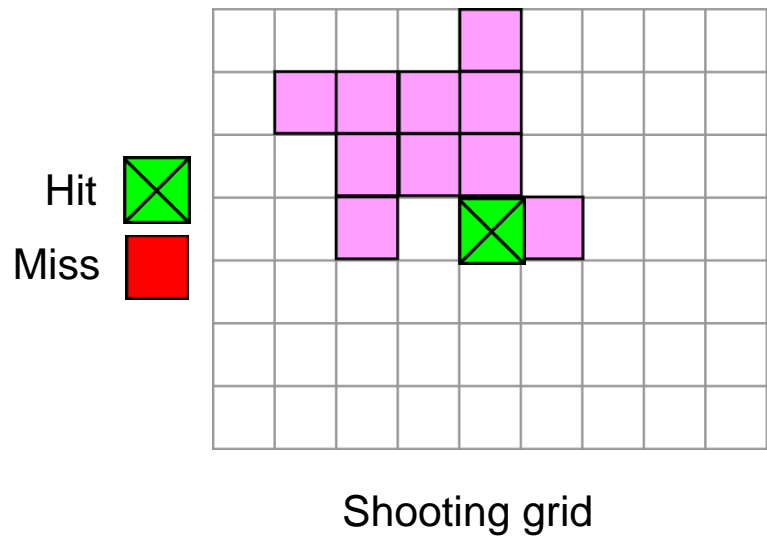
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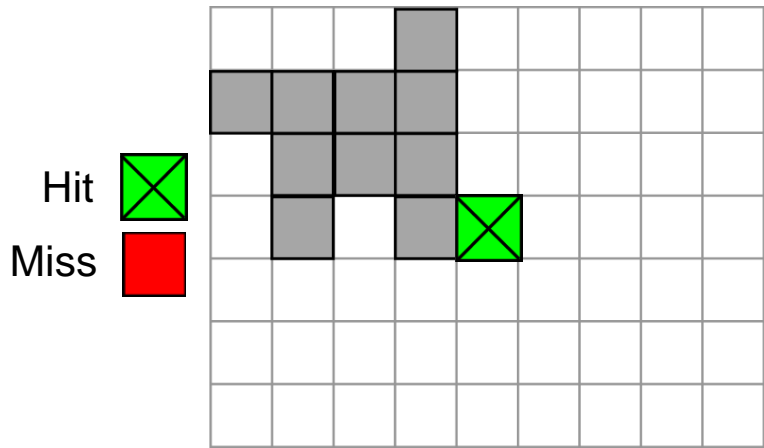
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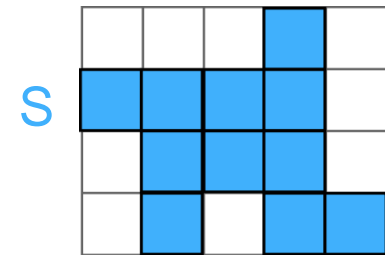
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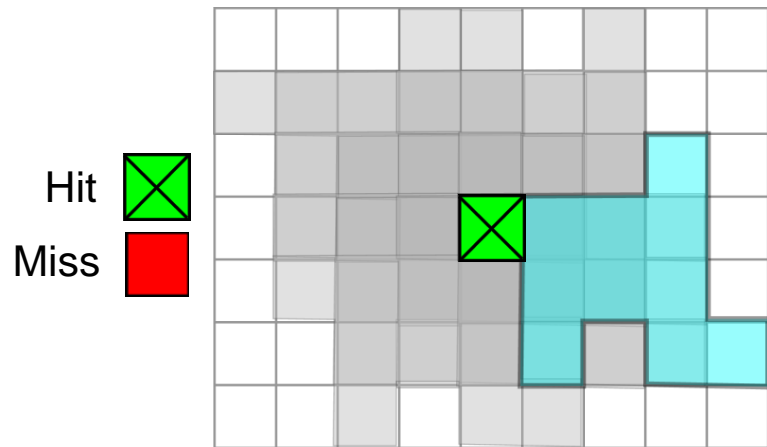
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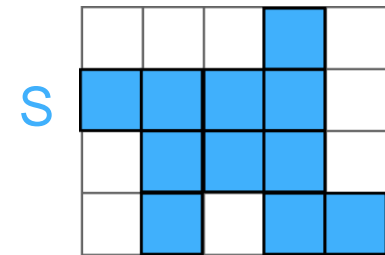


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Shooting grid

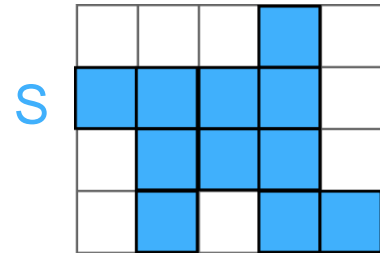
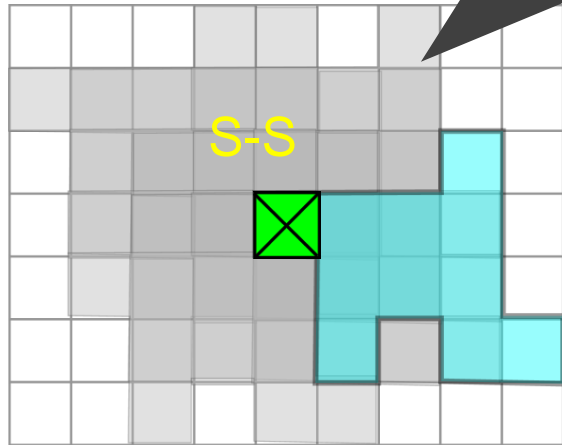
n positions are possible...



We assume that the ship has been **hit** one time by shooting at coordinates $(0,0)$

Never shoot outside from S-S : otherwise, it's a miss!

Hit 
Miss 

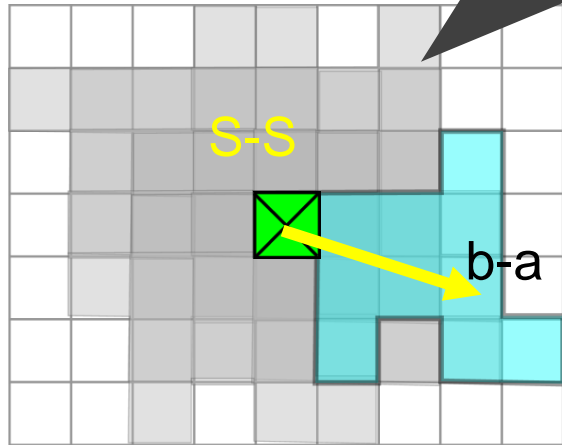
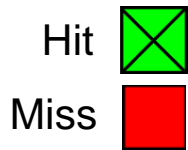


Shooting grid

n positions are possible...

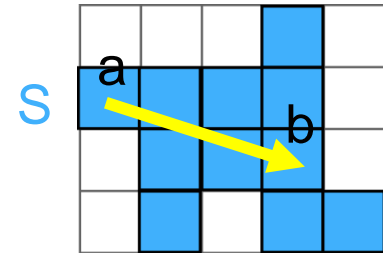
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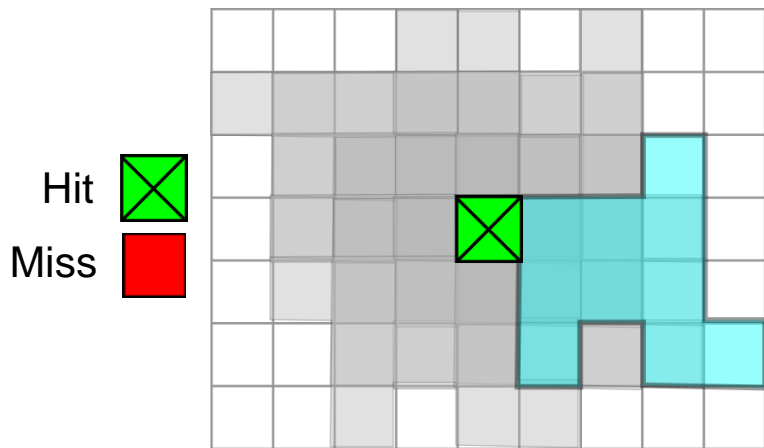


Shooting grid

n positions are possible...

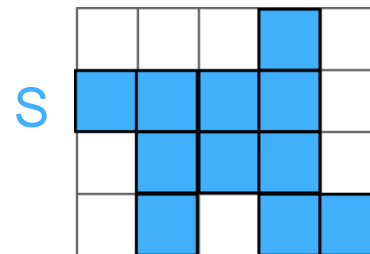


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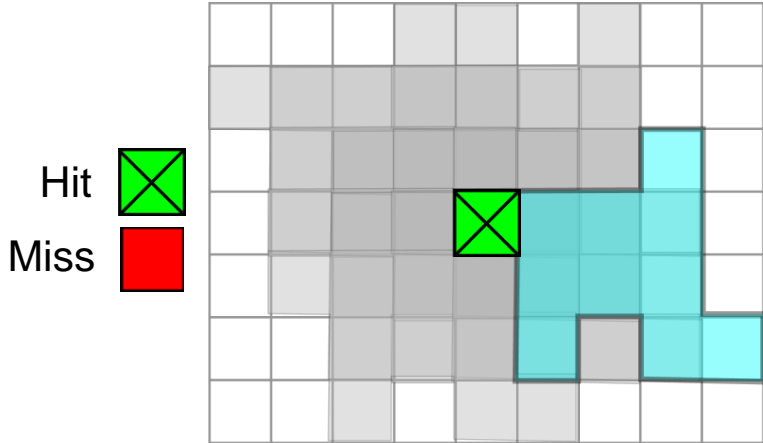
Shooting grid

n positions are possible...



Now, the goal is :
sink the ship with the **fewer number of shots** as possible

We assume that the ship has been **hit** one time by shooting at coordinates **(0,0)**



Shooting grid

n positions are possible...



translation:
sink the ship with the **fewer number of misses** as possible

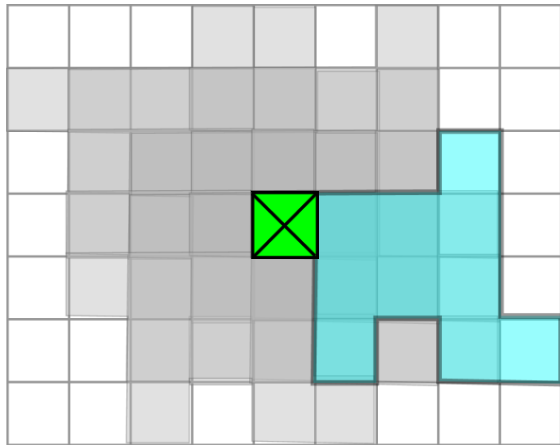
Now, the goal is :
sink the ship with the **fewer number of shots** as possible

We assume that the ship has been **hit** one time by shooting at coordinates **(0,0)**

Find the position of the ship with the fewer number of misses as possible

translation:
sink the ship with the fewer number of misses as possible

Hit 
Miss 



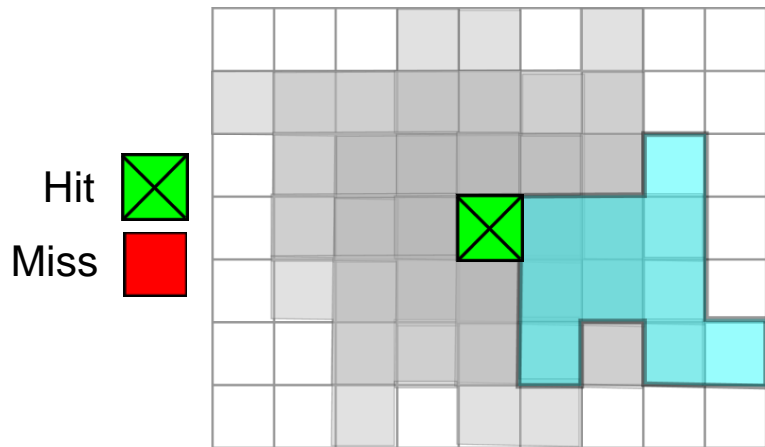
Shooting grid

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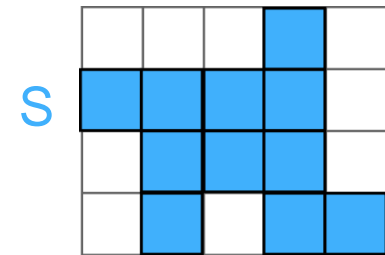
We assume that the ship has been hit one time by shooting at coordinates (0,0)

Find the position of the ship
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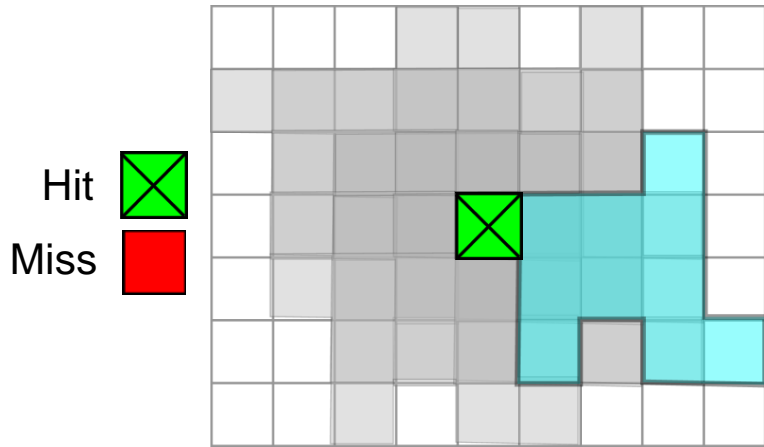
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n positions are possible...

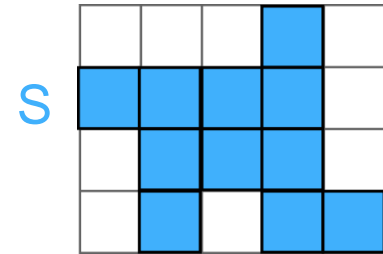


We assume that the ship has been hit one time by shooting at coordinates (0,0)

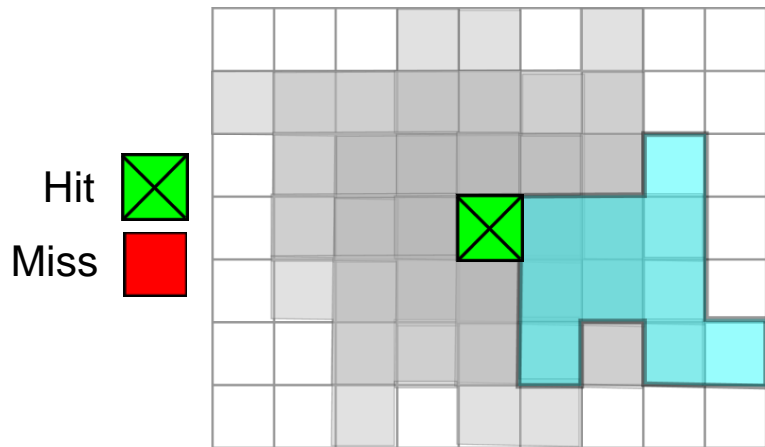
Find the position of the ship with the fewer number of misses as possible



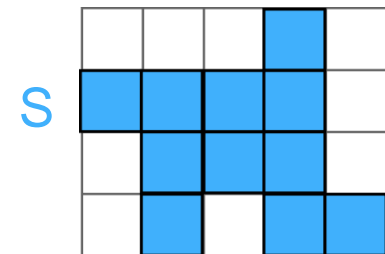
Shooting grid



Find the position of the ship
with the fewer number of
misses as possible



Shooting grid



Design a shooting algorithm to find the position of the ship
with the fewer number of misses...

Plan

I

Problem Statement

II

Shooting Algorithms

III

Results

IV

Proofs

Plan

I

Problem Statement

II

Shooting Algorithms

III

Results

IV

Proofs

Given a shape, our goal is to design a shooting algorithm
to find the position of the ship
with the fewer number of misses...

Given a shape, our goal is to design a **shooting algorithm**
to **find the position of the ship**
with the **fewer number of misses...**

2 different ways to
model an algorithm

With **shooting rules**
(for simple algorithms)

With a **decision tree**
(when there are many cases)



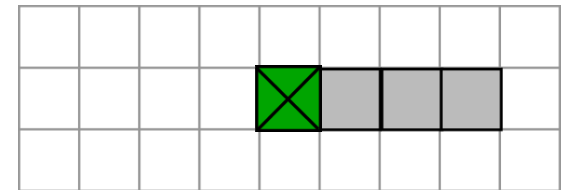
Shape of the ship

$$n=|S|=4$$



Shape of the ship

$$n=|S|=4$$



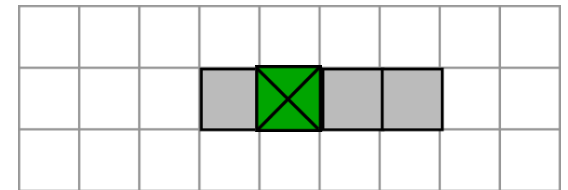
Shooting grid

4 positions are possible...



Shape of the ship

$$n=|S|=4$$



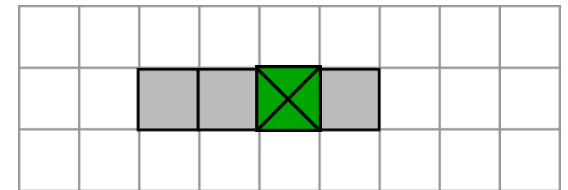
Shooting grid

4 positions are possible...



Shape of the ship

$$n=|S|=4$$



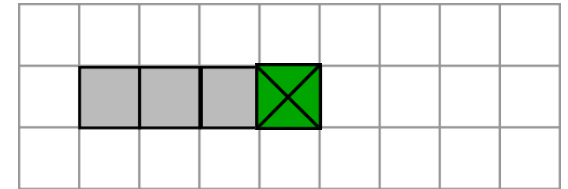
Shooting grid

4 positions are possible...



Shape of the ship

$$n=|S|=4$$



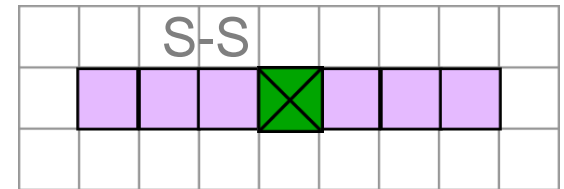
Shooting grid

4 positions are possible...



Shape of the ship

$$n=|S|=4$$



Shooting grid

4 positions are possible...

Shoot (1,0) ?

Shoot (2,0) ?

Shoot (3,0) ?

Shoot (-1,0) ?

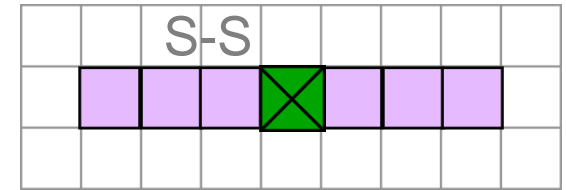
Shoot (-2,0) ?

Shoot (-3,0) ?



Shape of the ship

$$n=|S|=4$$



Shooting grid

4 positions are possible...

Shoot (1,0) ?

Shoot (2,0) ?

Shoot (3,0) ?

Shoot (-1,0) ?

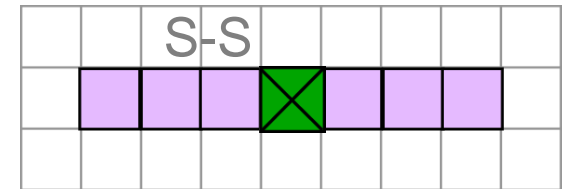
Shoot (-2,0) ?

Shoot (-3,0) ?



Shape of the ship

$$n=|S|=4$$



Shooting grid

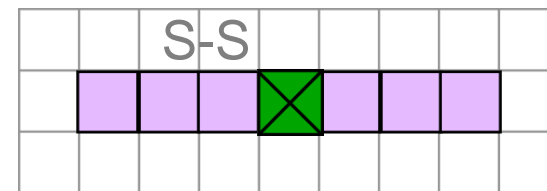
4 positions are possible...

What is the best strategy ?



Shape of the ship

$$n=|S|=4$$



Shooting grid

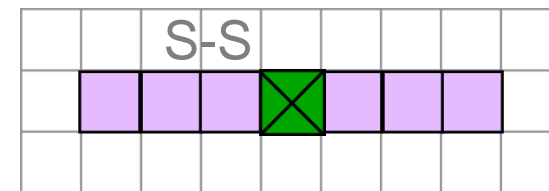
4 positions are possible...

What is the best strategy ?



Shape of the ship

$$n=|S|=4$$



Shooting grid

4 positions are possible...

A shooting algorithm:

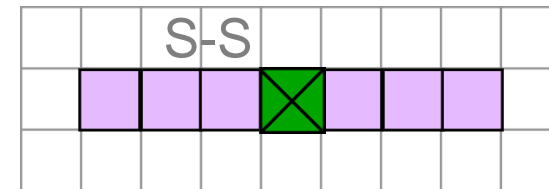
Shoot on the left of the previous shot until having a miss.
Then the position of the ship is known.

What is the best strategy ?



Shape of the ship

$$n=|S|=4$$



Shooting grid

4 positions are possible...

A shooting algorithm:

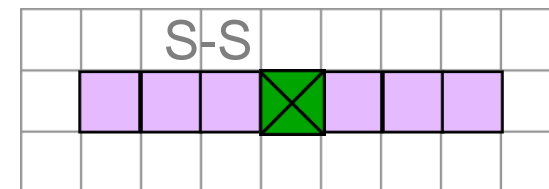
Shoot on the left of the previous shot until having a miss.
Then the position of the ship is known.

What is the best strategy ?



Shape of the ship

$$n = |S| = 4$$



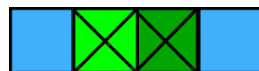
Shooting grid

4 positions are possible...

A shooting algorithm:

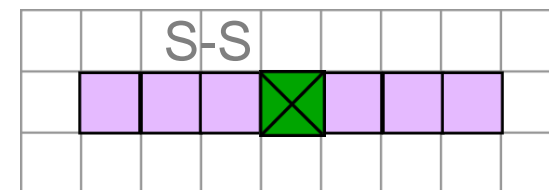
Shoot on the left of the previous shot until having a miss.
Then the position of the ship is known.

What is the best strategy ?



Shape of the ship

$$n=|S|=4$$

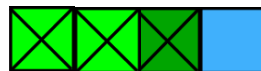


Shooting grid

4 positions are possible...

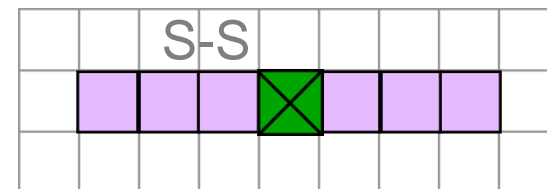
A shooting algorithm:
Shoot on the left of the previous shot until having a miss.
Then the position of the ship is known.

What is the best strategy ?



Shape of the ship

$$n=|S|=4$$

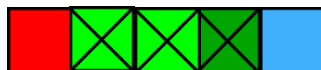


Shooting grid

4 positions are possible...

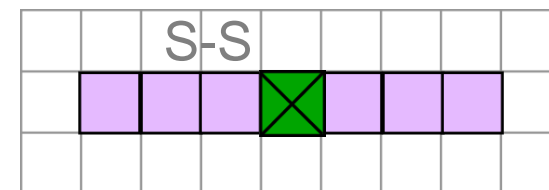
A shooting algorithm:
Shoot on the left of the previous shot until having a miss.
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What is the best strategy ?



Shape of the ship

$$n=|S|=4$$



Shooting grid

4 positions are possible...

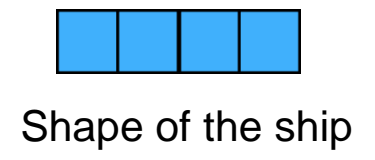
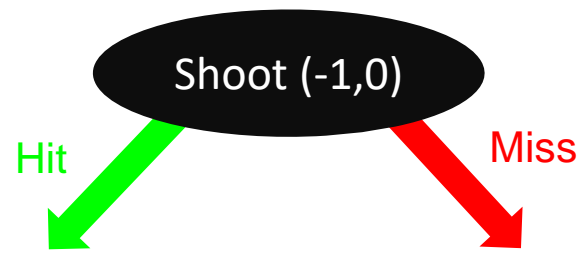
A shooting algorithm:
Shoot on the left of the previous shot until having a miss.
Then the position of the ship is known.

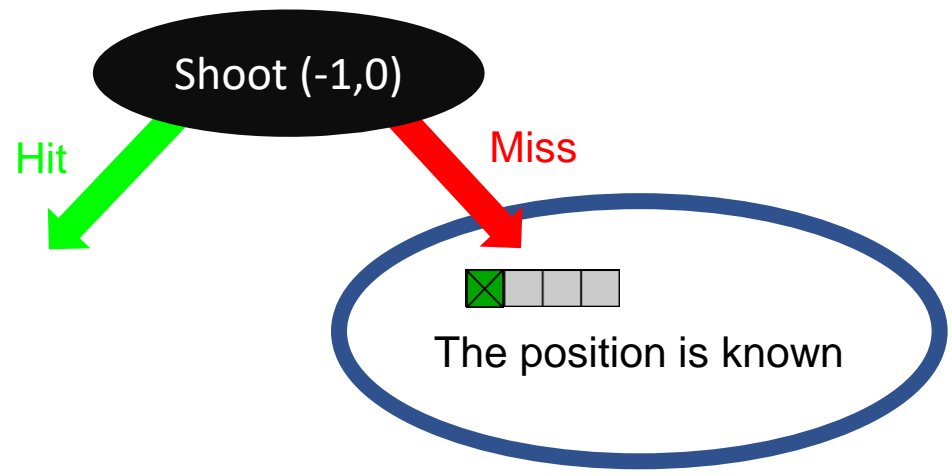
What is the best strategy ?

Shoot (-1,0)

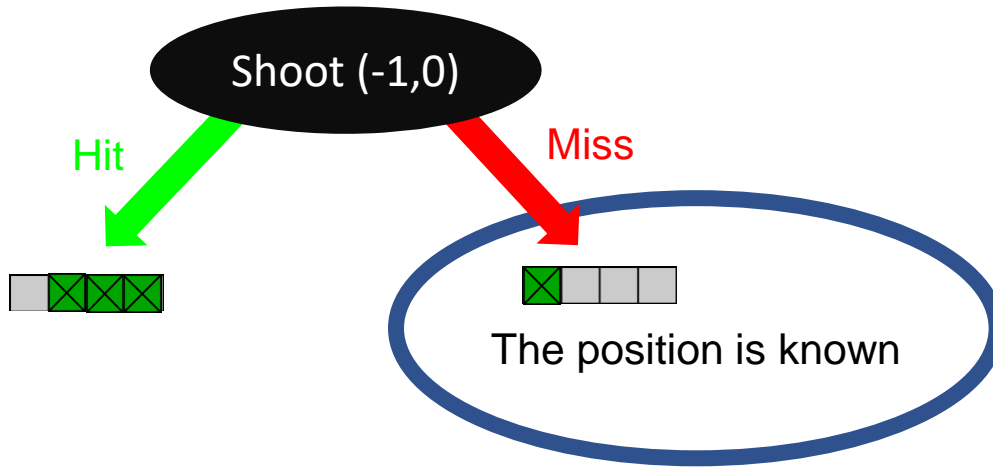


Shape of the ship

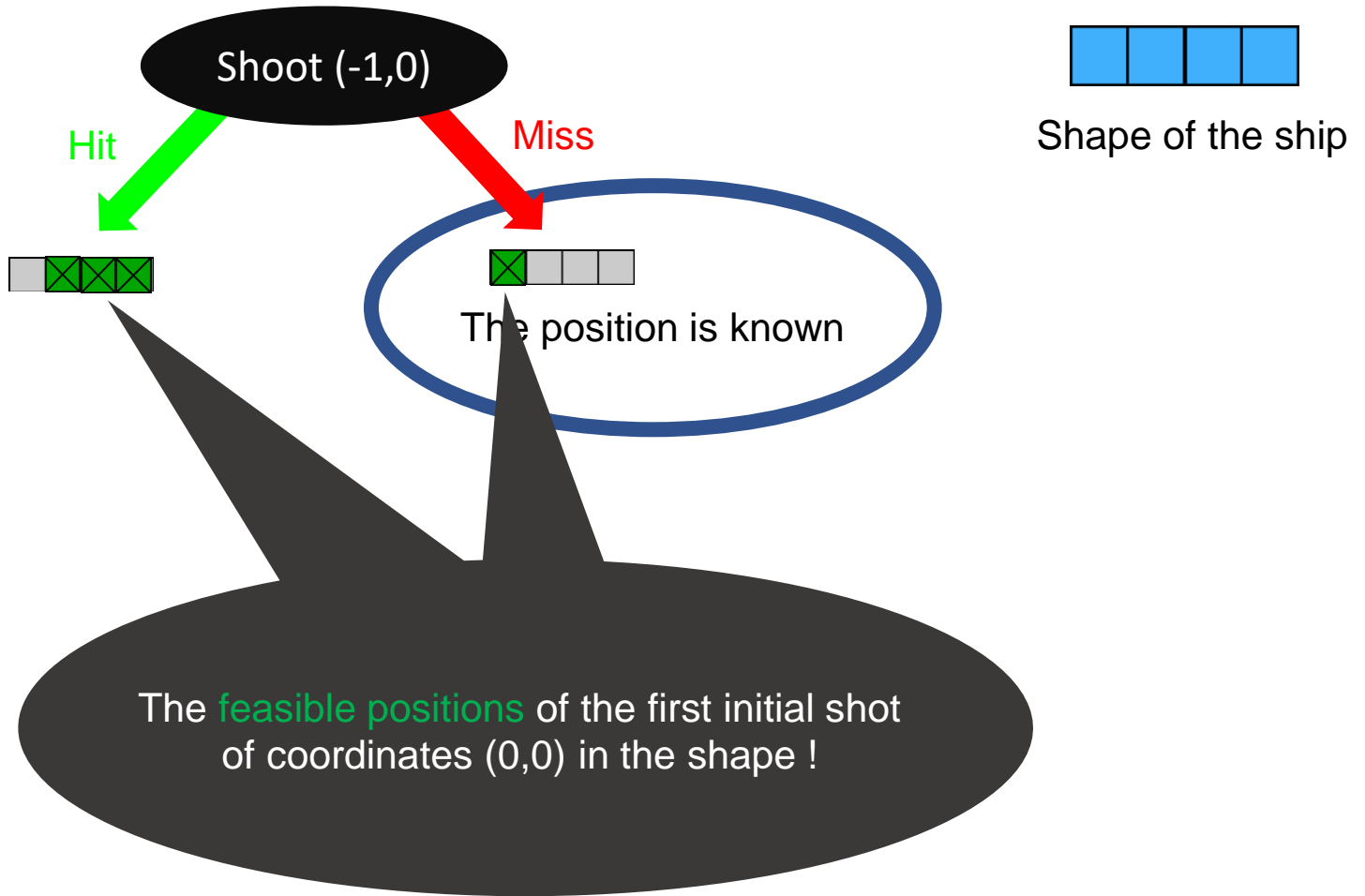


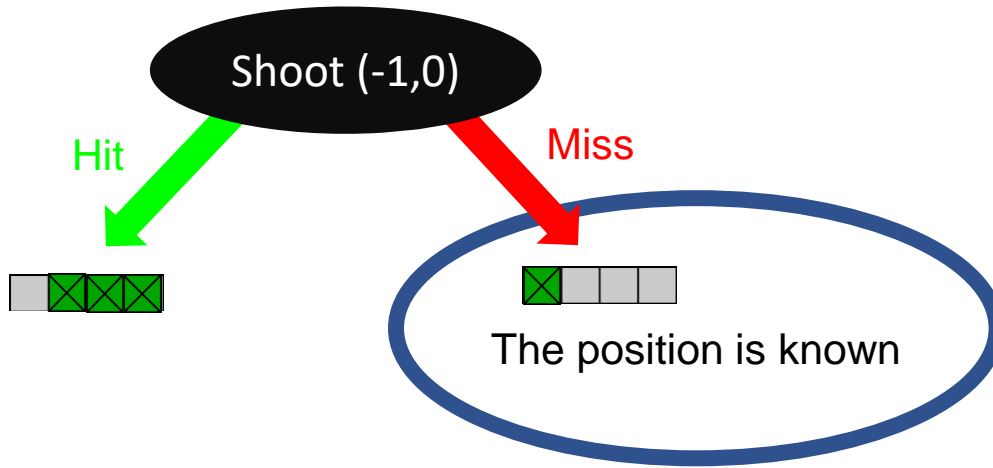


Shape of the ship

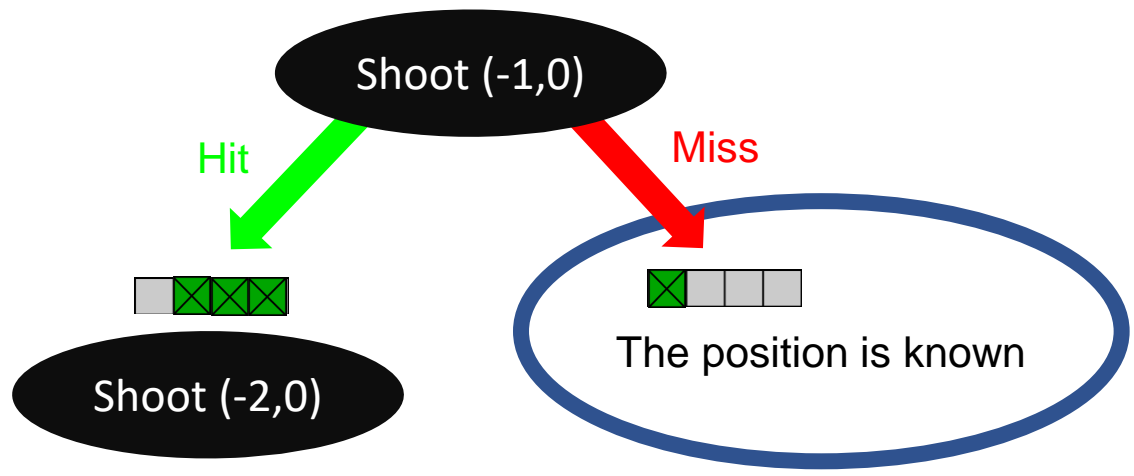


Shape of the ship

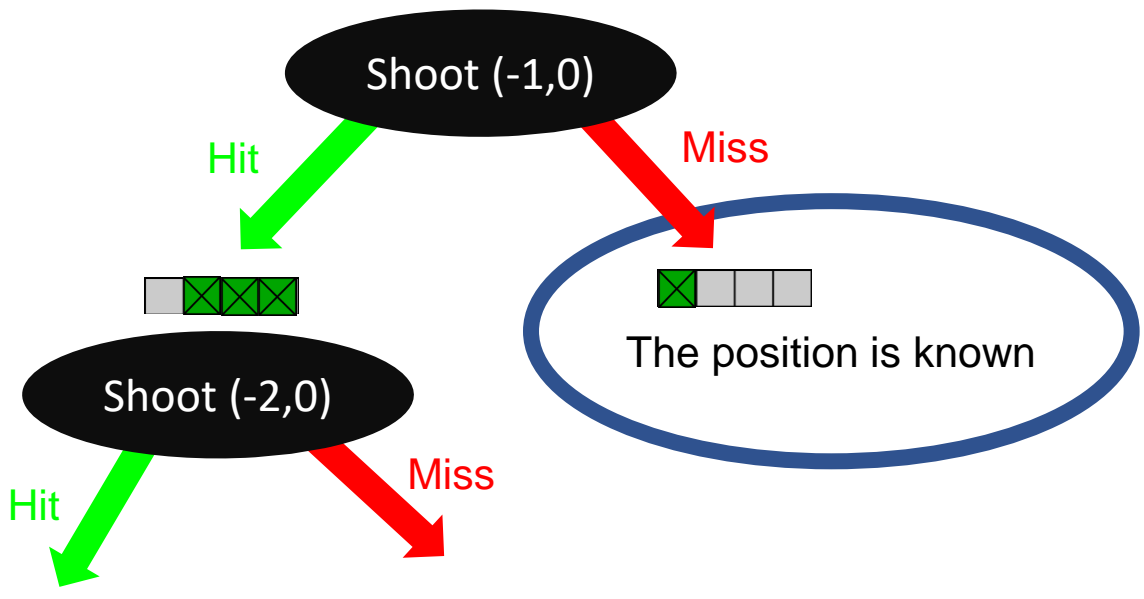




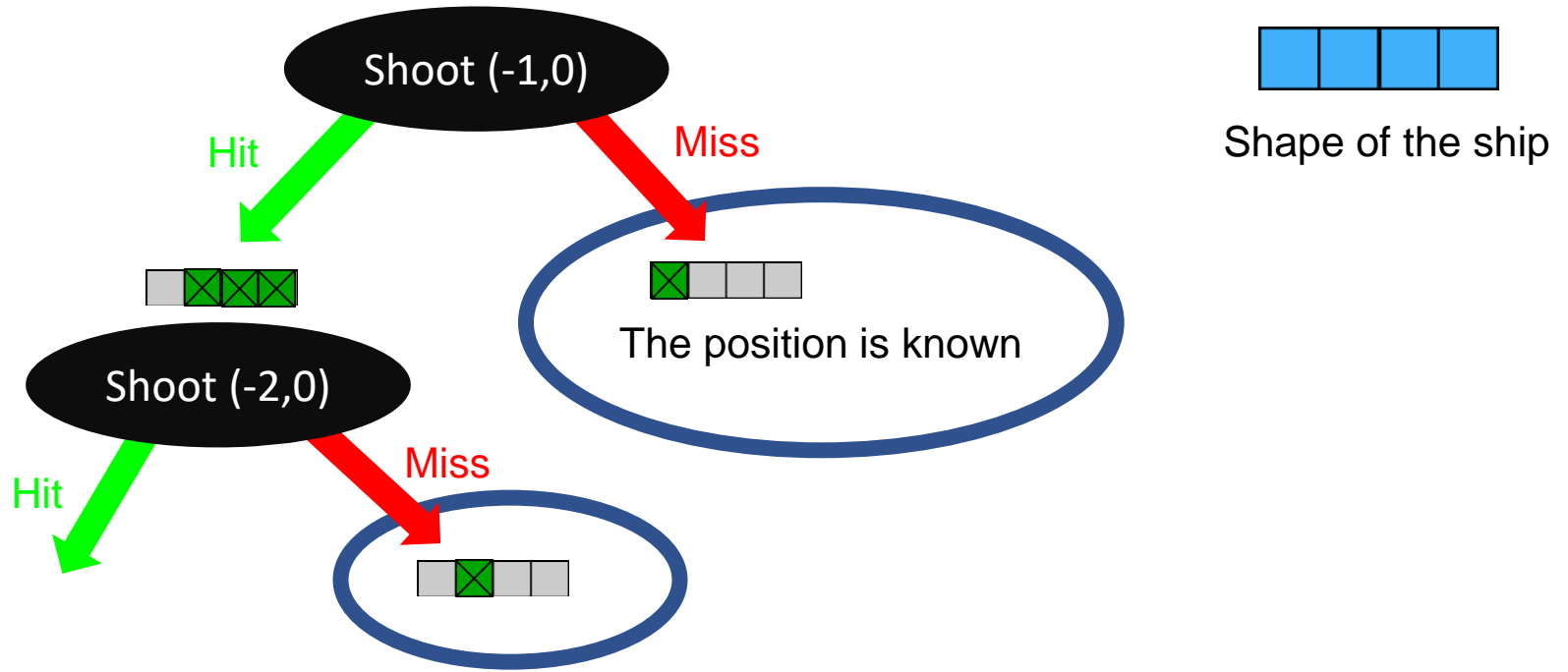
Shape of the ship



Shape of the ship

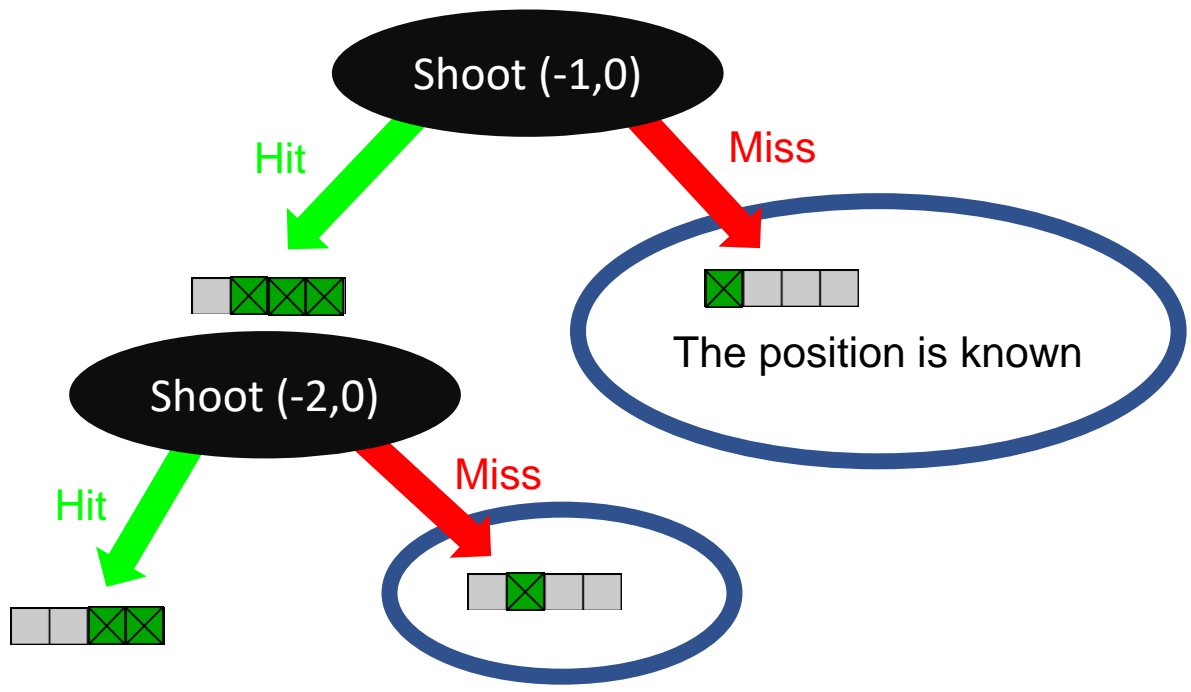


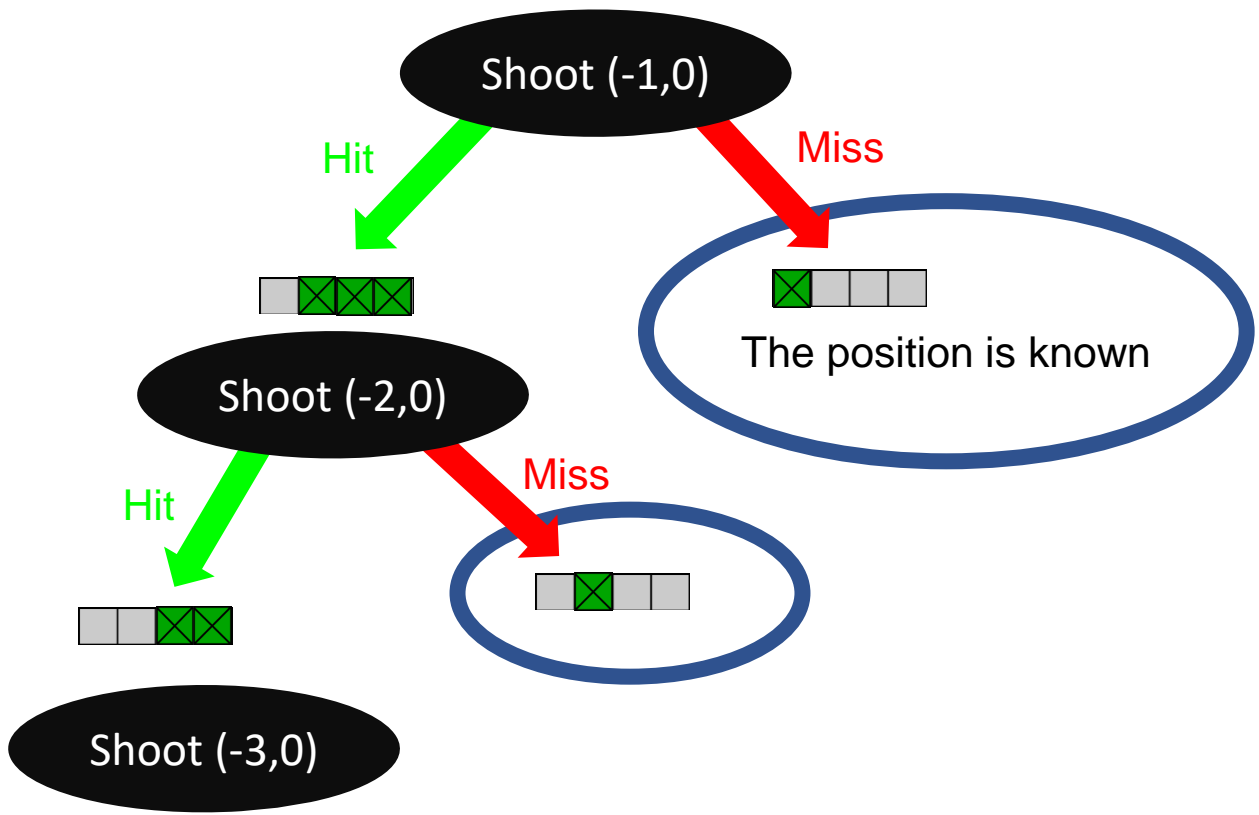
Shape of the ship





Shape of the ship





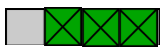
Shape of the ship



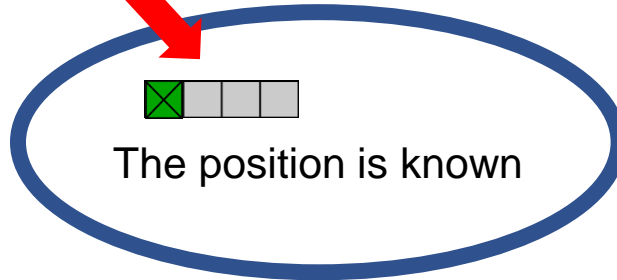
Shape of the ship

Shoot (-1,0)

Hit



Miss

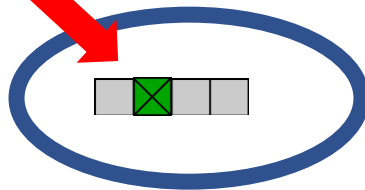


Shoot (-2,0)

Hit



Miss

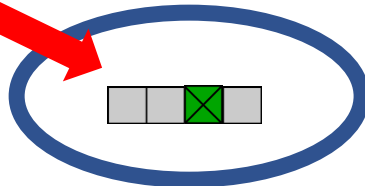


Shoot (-3,0)

Hit

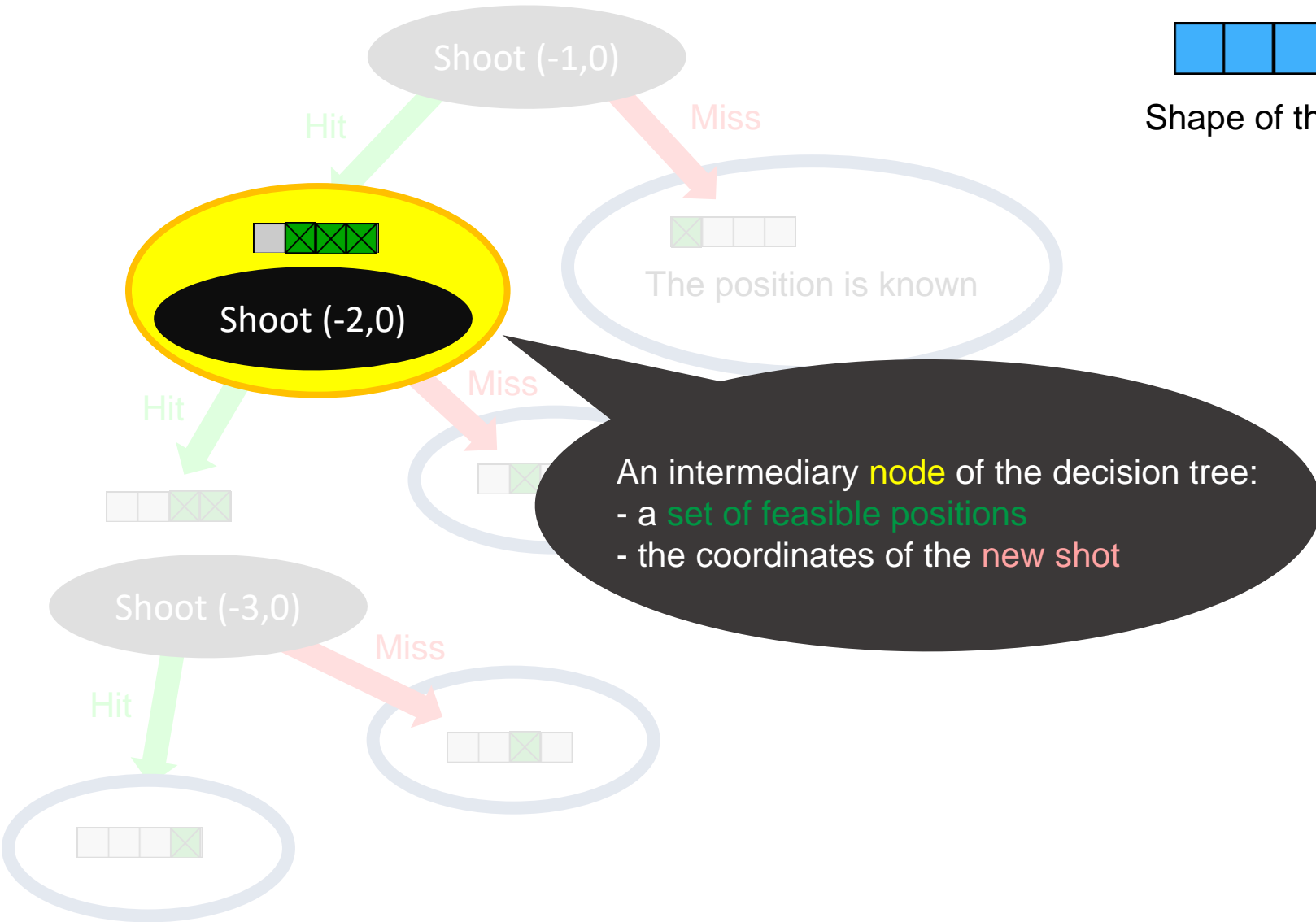


Miss





Shape of the ship





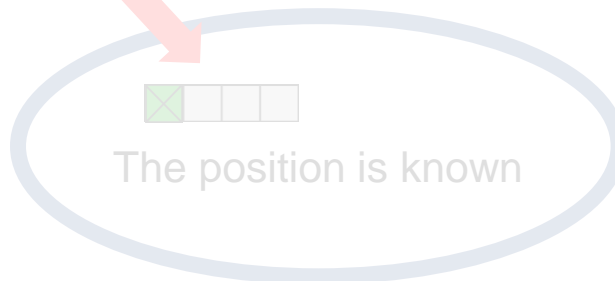
Shape of the ship

Shoot (-1,0)

Hit



Miss

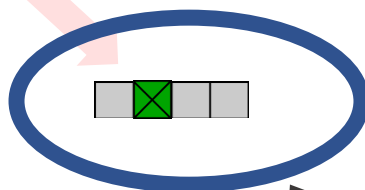


Shoot (-2,0)

Hit



Miss

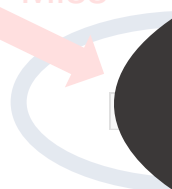


Shoot (-3,0)

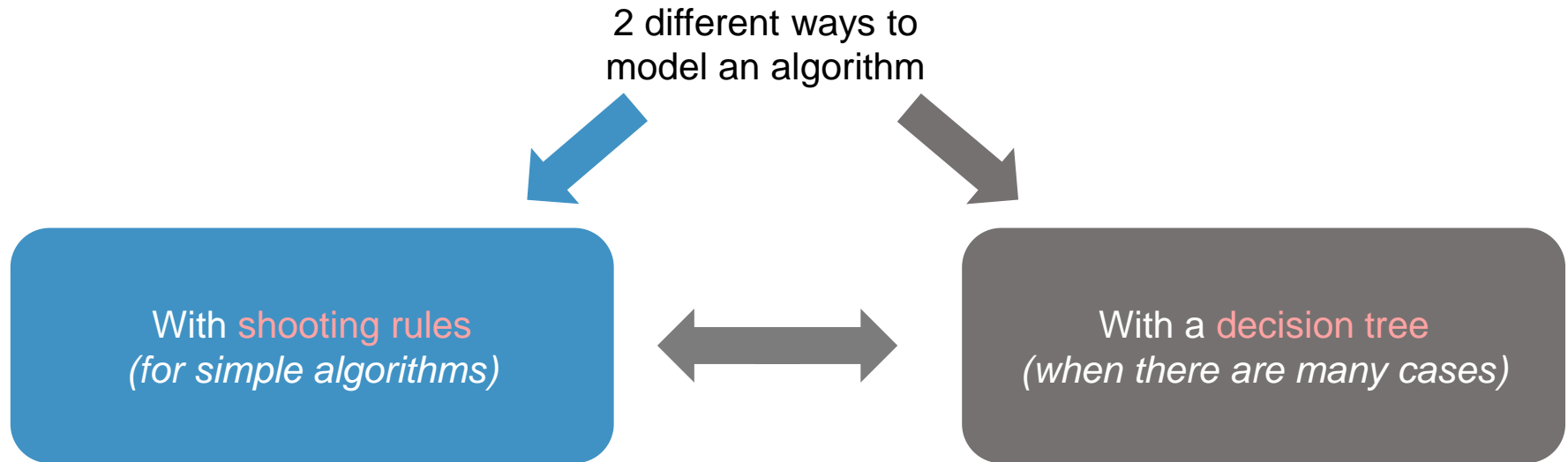
Hit

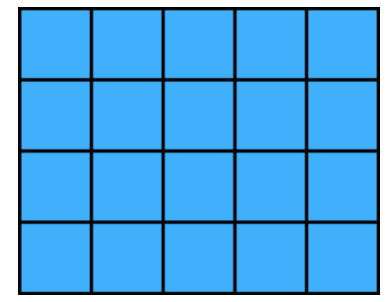


Miss

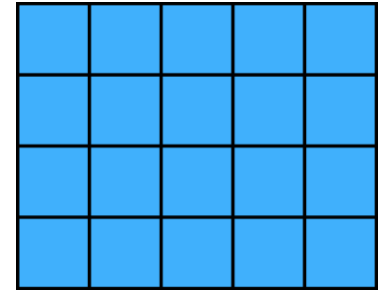


We have a **leaf** when it remains only ONE **feasible positions**.





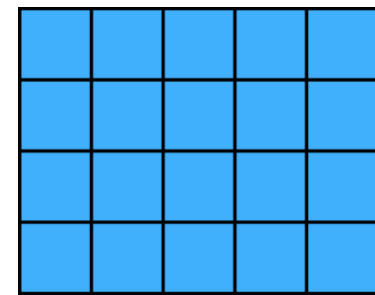
Shape of the ship



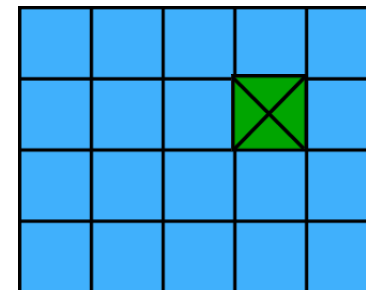
Shape of the ship

What is the best strategy ?

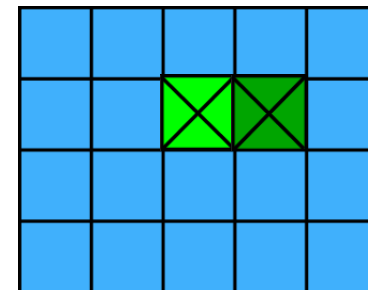
1) Shoot on the left of the previous shot until having a miss.



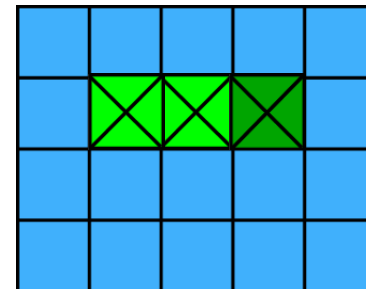
1) Shoot on the left of the previous shot until having a miss.



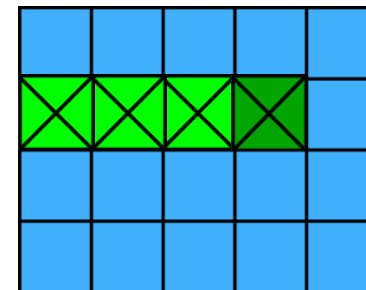
1) Shoot on the left of the previous shot until having a miss.



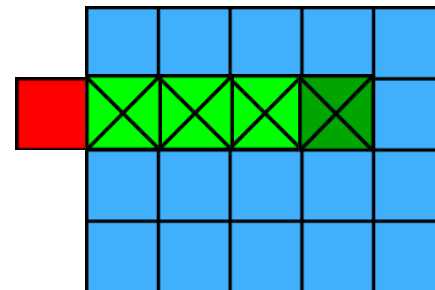
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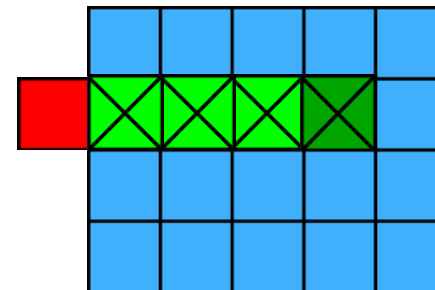
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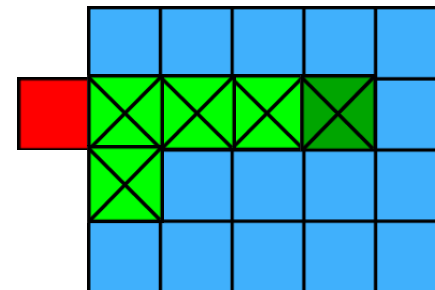
1) Shoot on the left of the previous shot until having a miss.



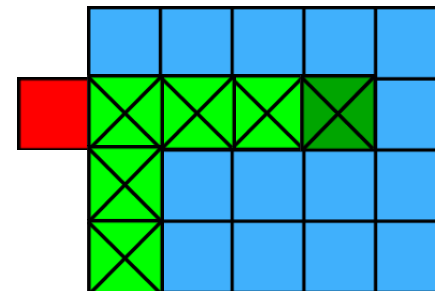
- 1) Shoot on the left of the previous shot until having a miss.
- 2) Then shoot down until having a miss.
Then the position of the ship is known.



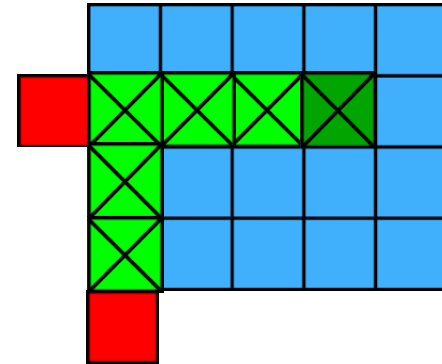
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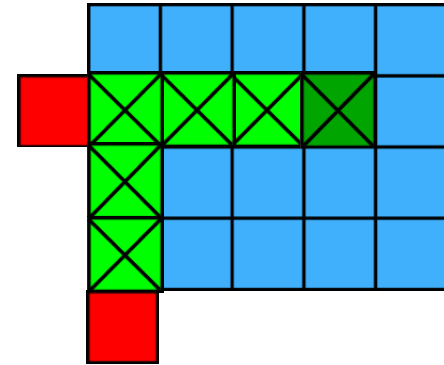
- 1) Shoot on the left of the previous shot until having a miss.
- 2) Then shoot down until having a miss.
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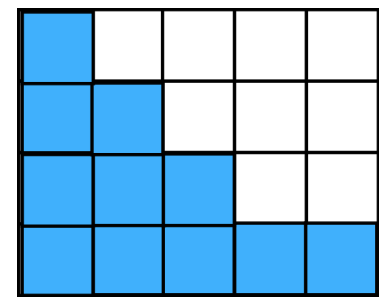
- 1) Shoot on the left of the previous shot until having a miss.
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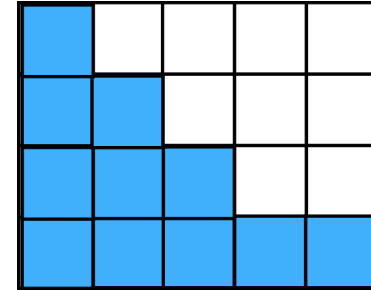


It allows to determine the position of the ship with **at most two misses**



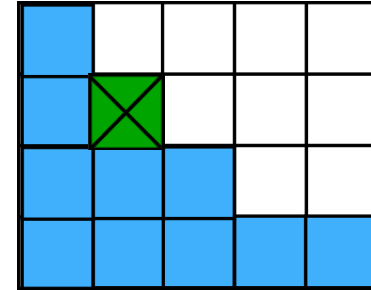
A new shape

The previous algorithm works: it determines the position with **at most 2 misses**.

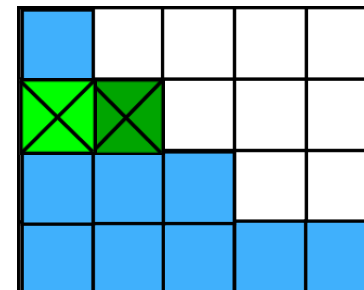


A new shape

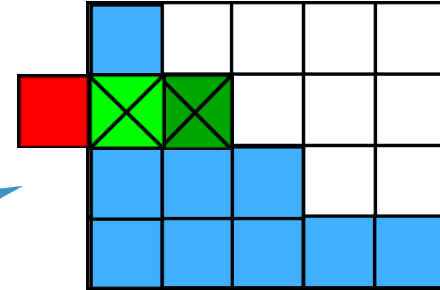
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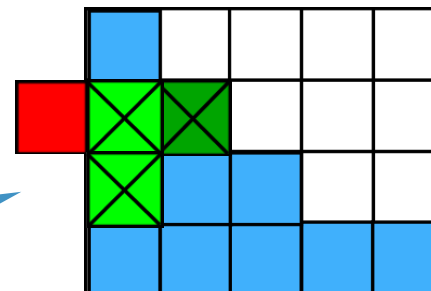
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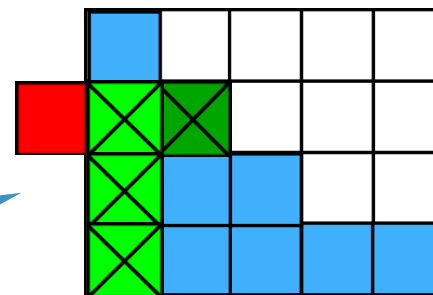
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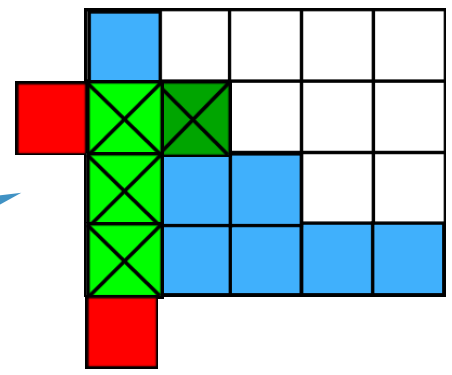
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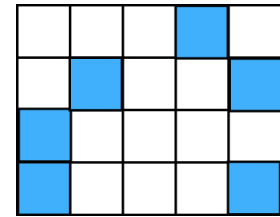


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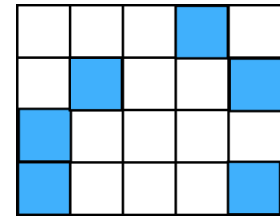
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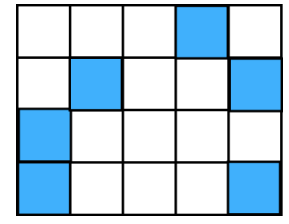
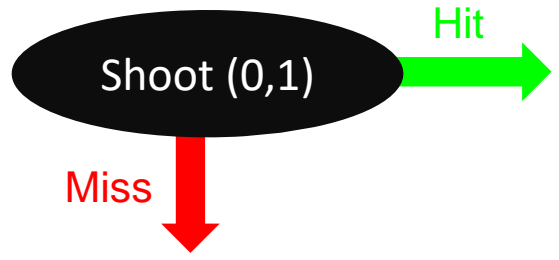


Shape

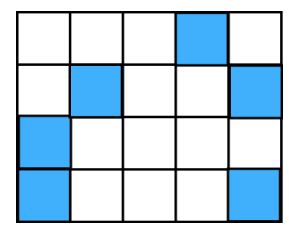
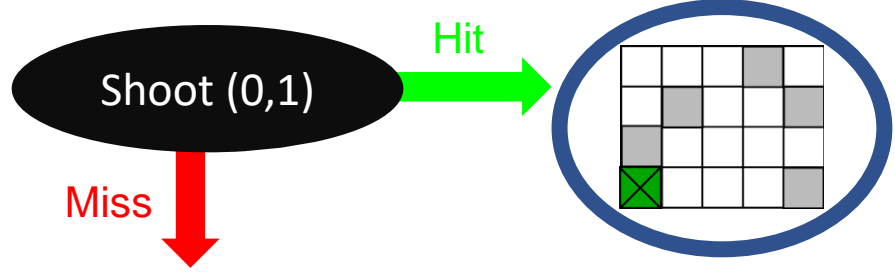
Shoot (0,1)



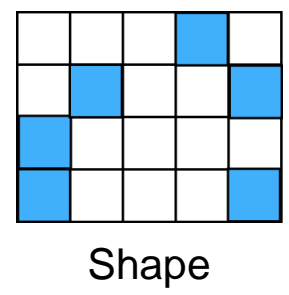
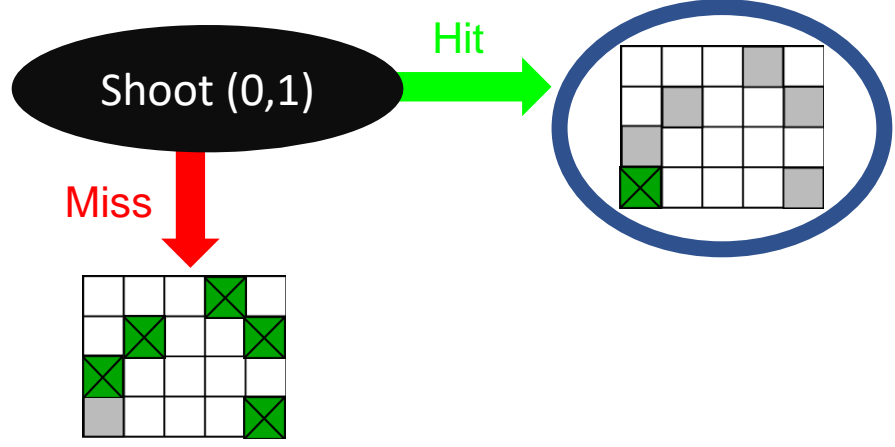
Shape



Shape

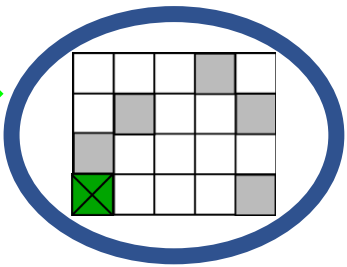


Shape

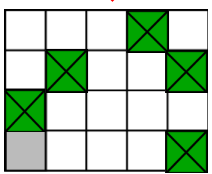


Shoot (0,1)

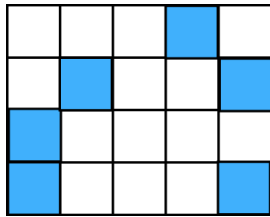
Hit



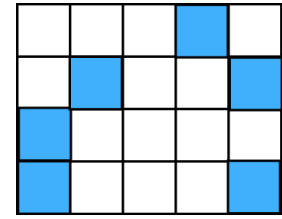
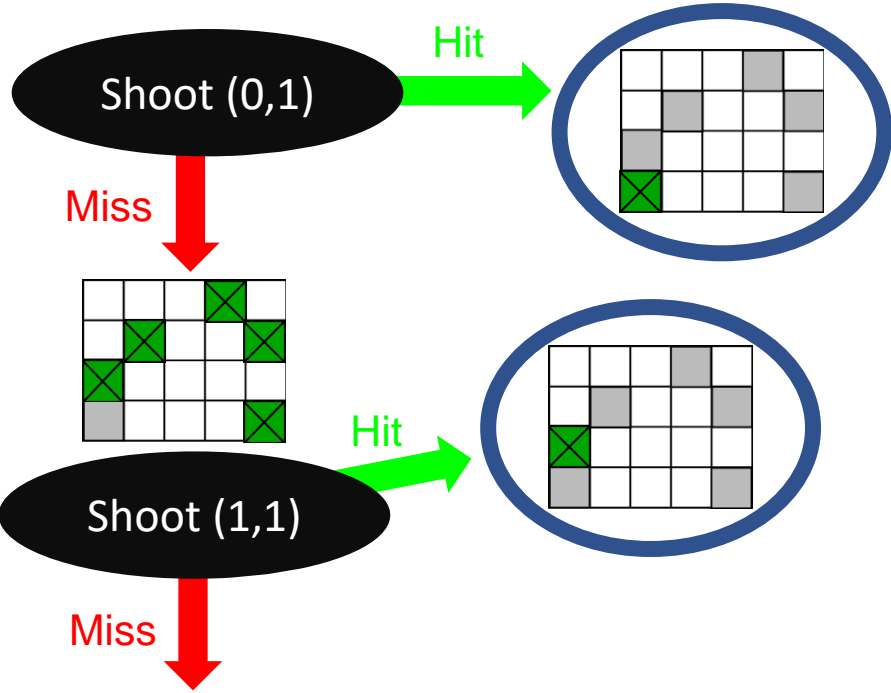
Miss



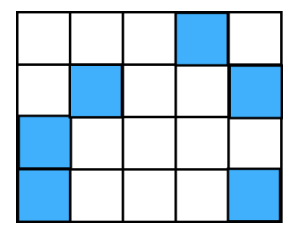
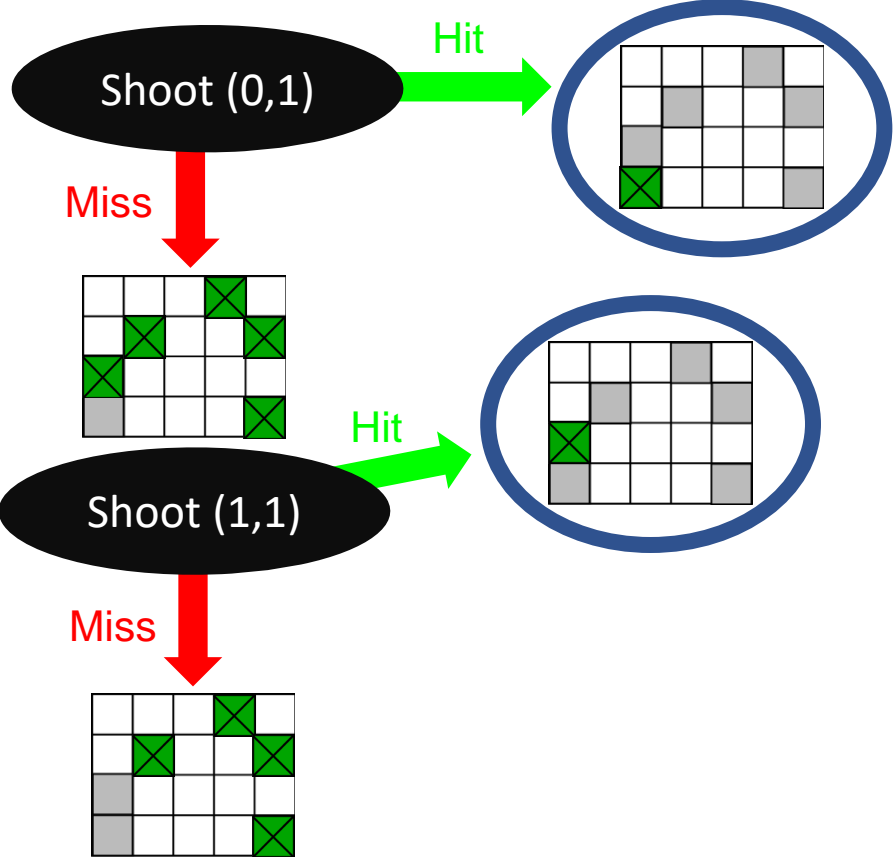
Shoot (1,1)



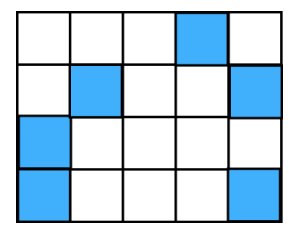
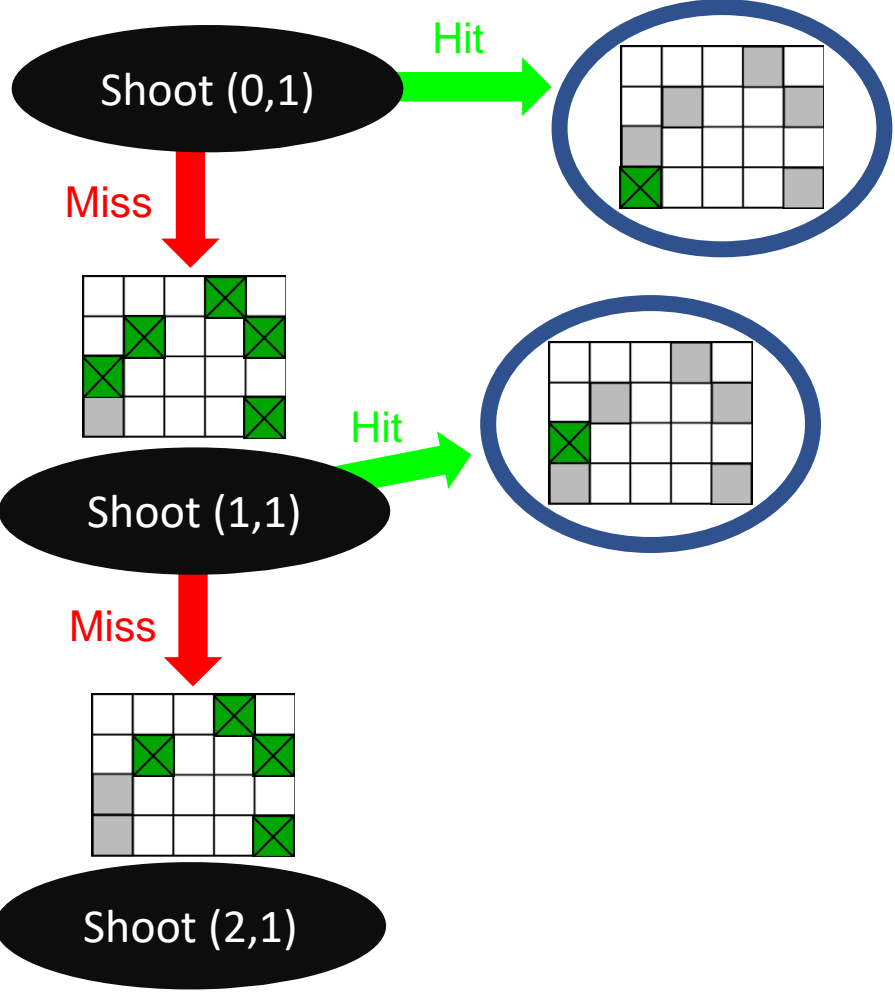
Shape



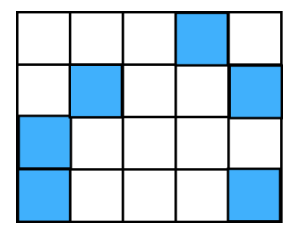
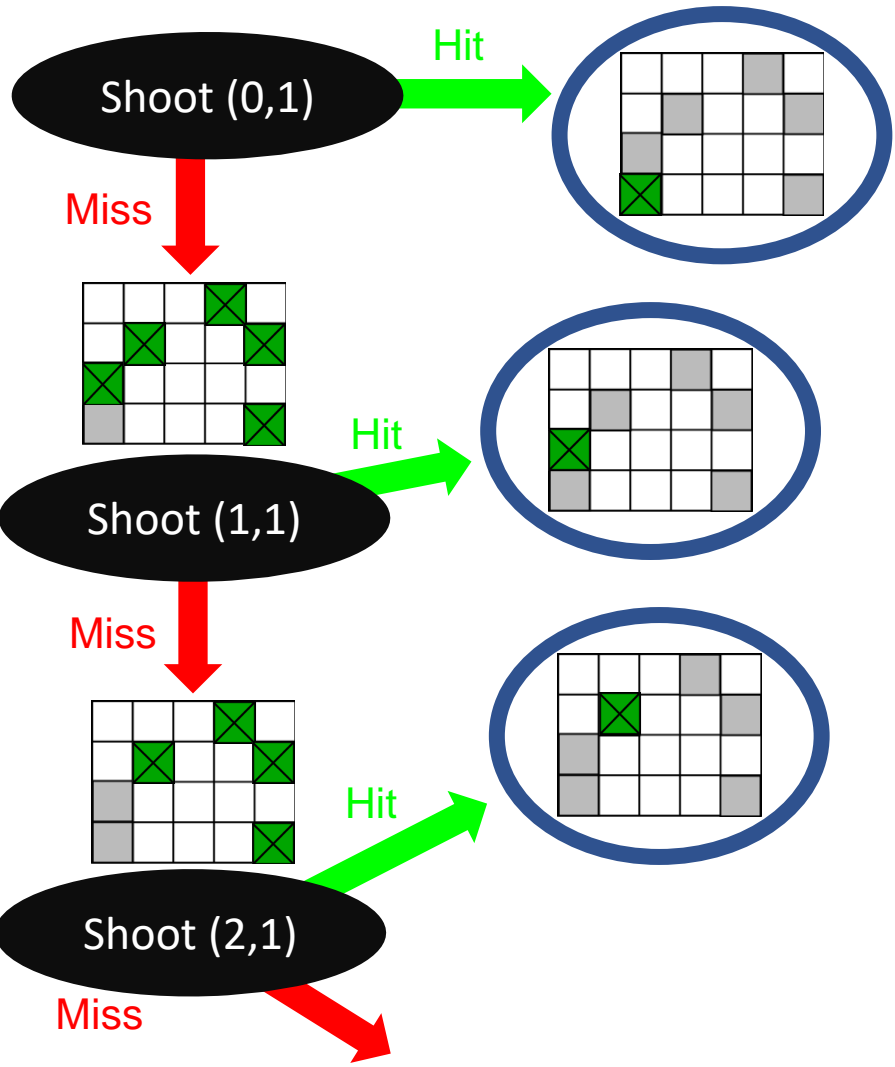
Shape



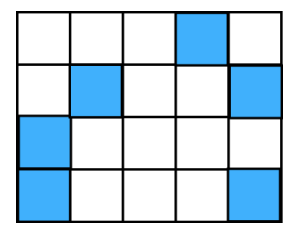
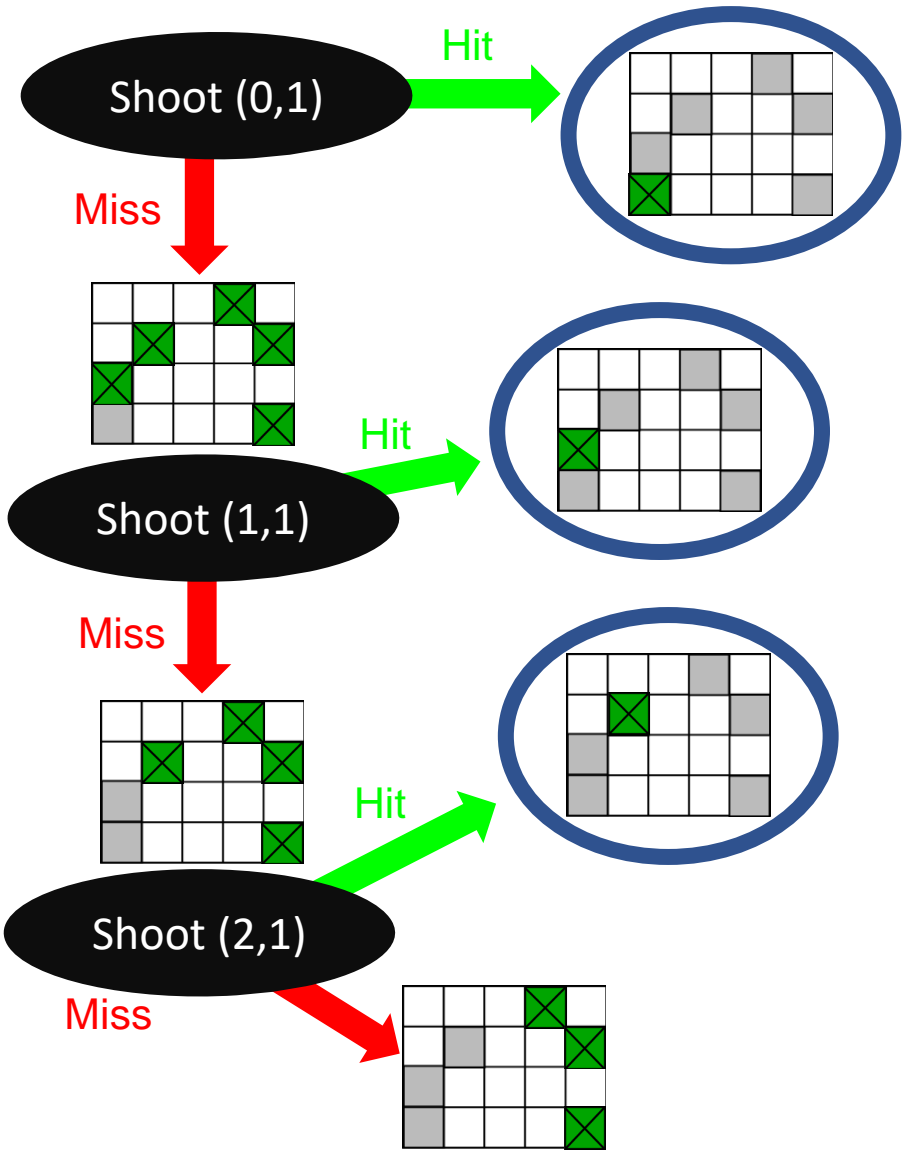
Shape



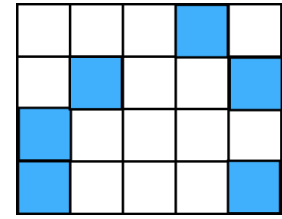
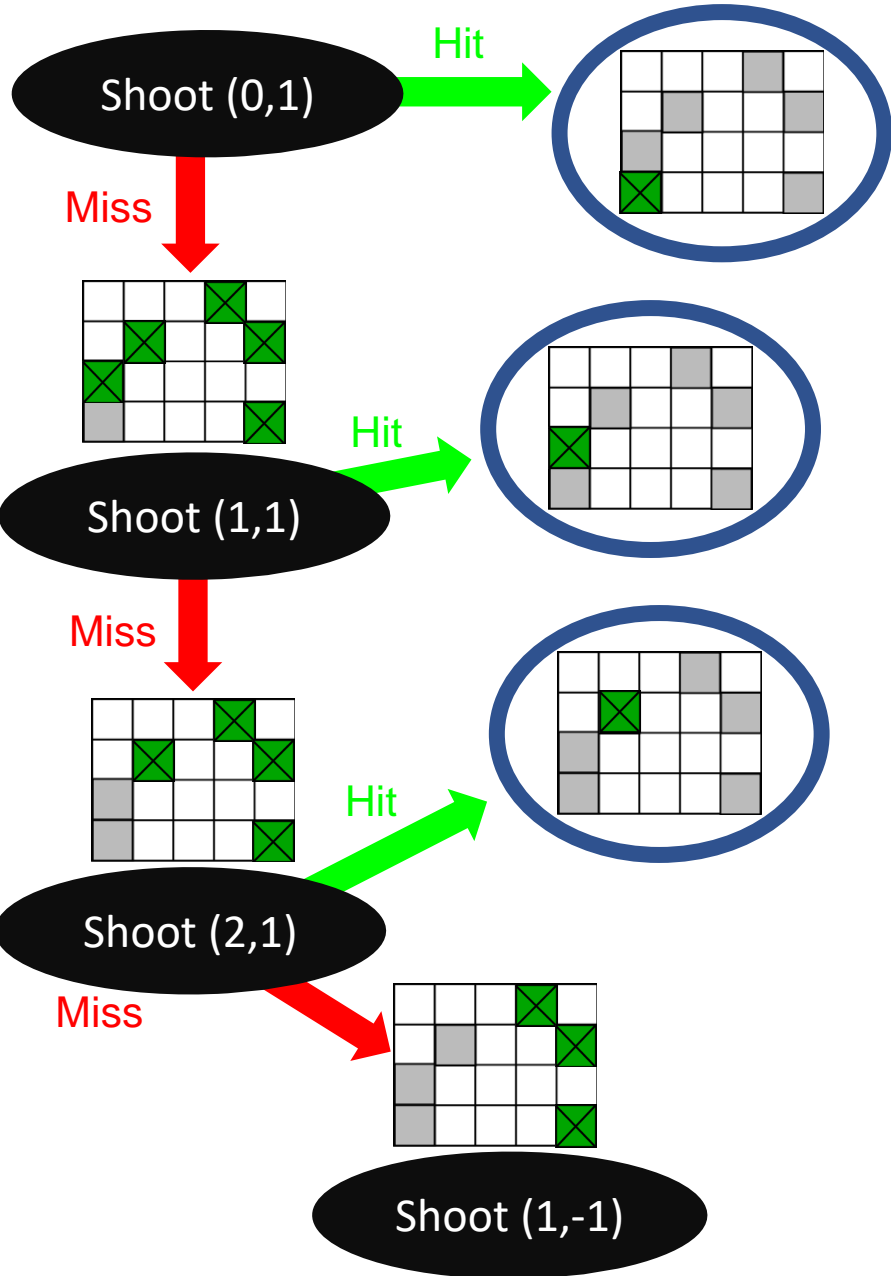
Shape



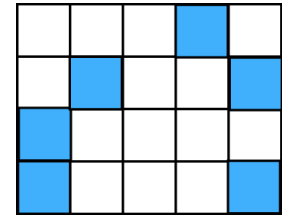
Shape



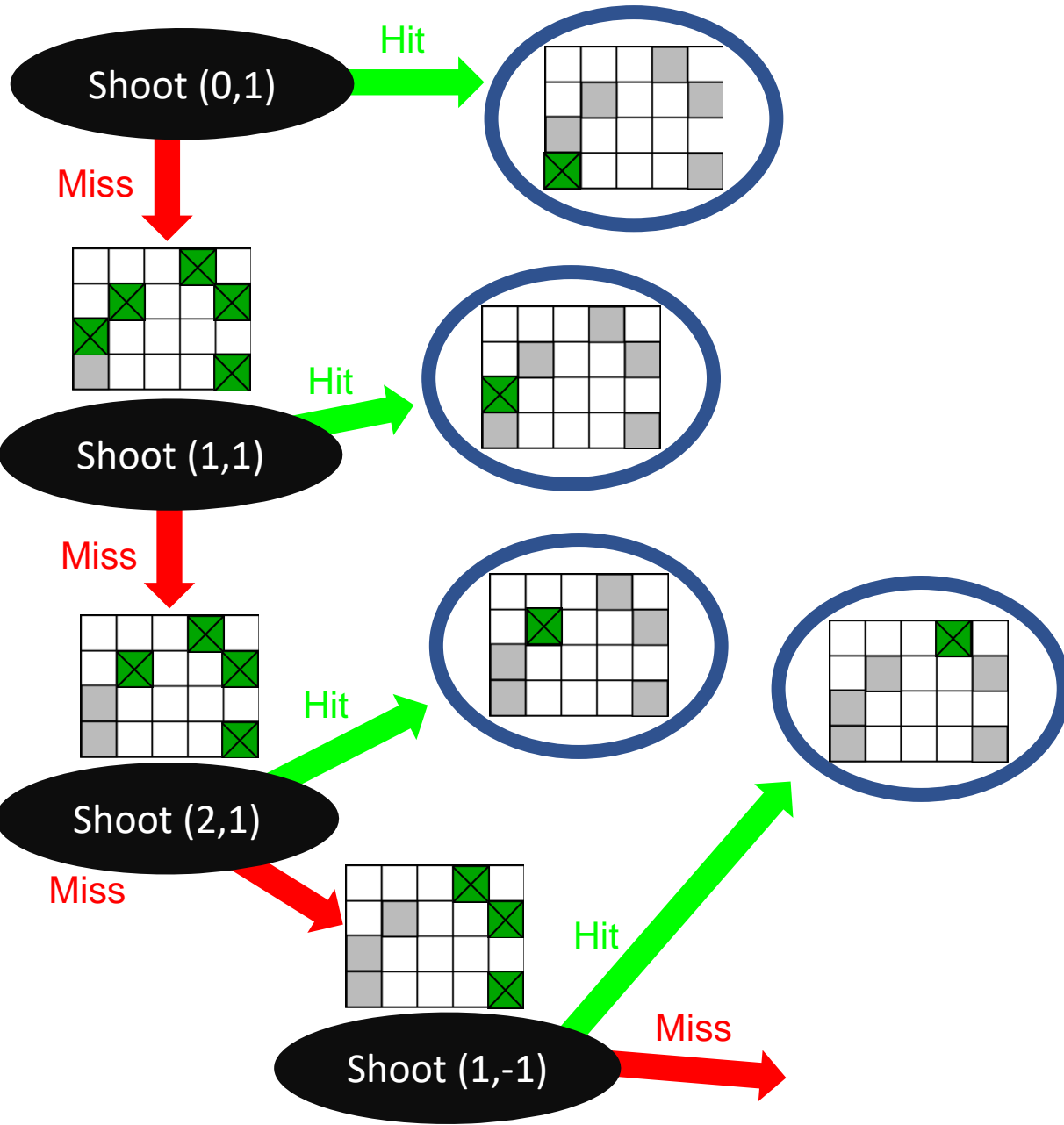
Shape

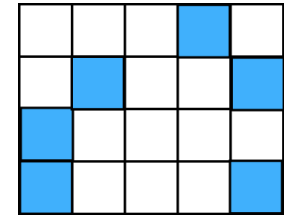


Shape

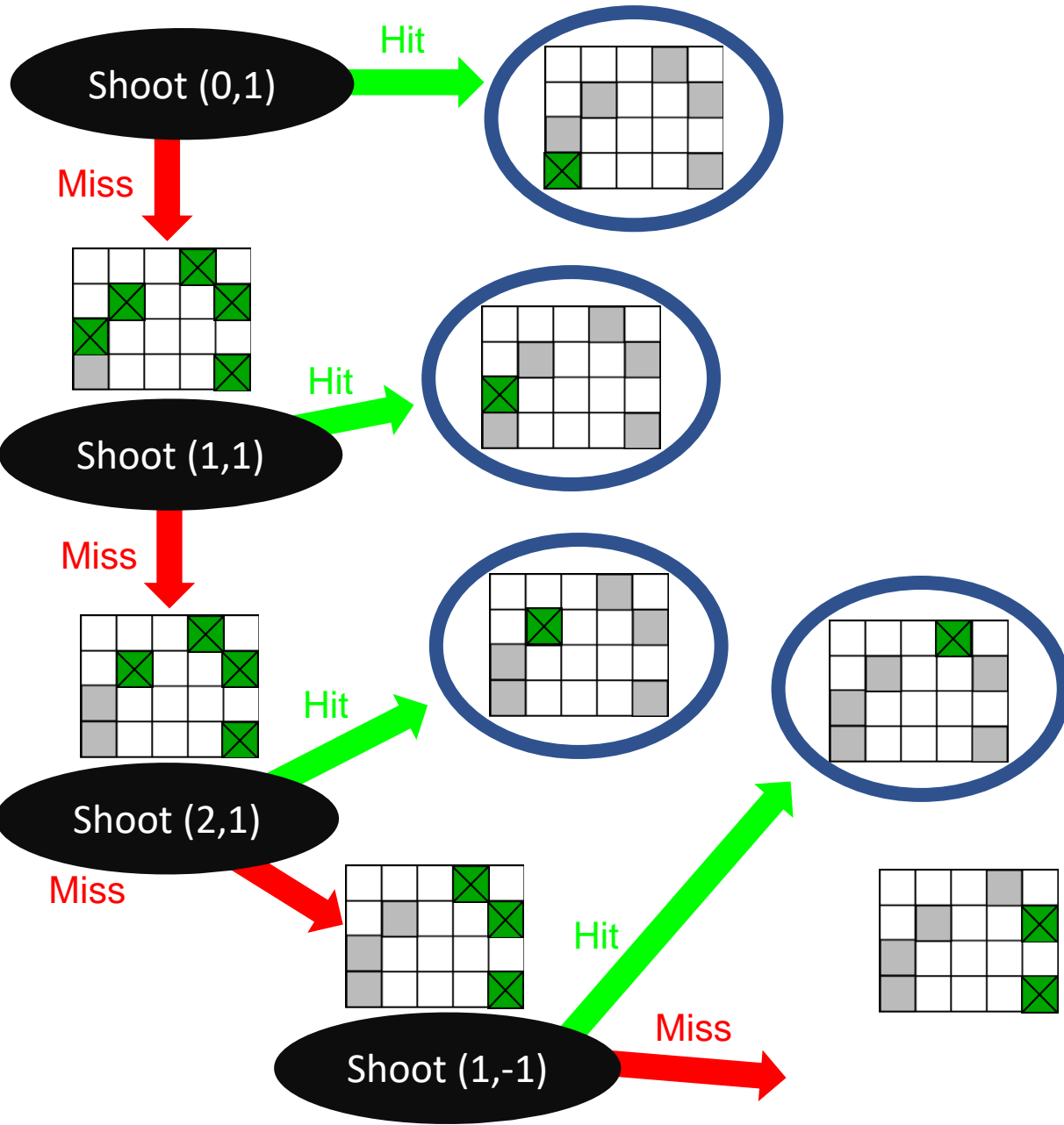


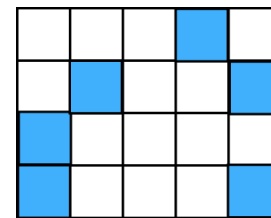
Shape



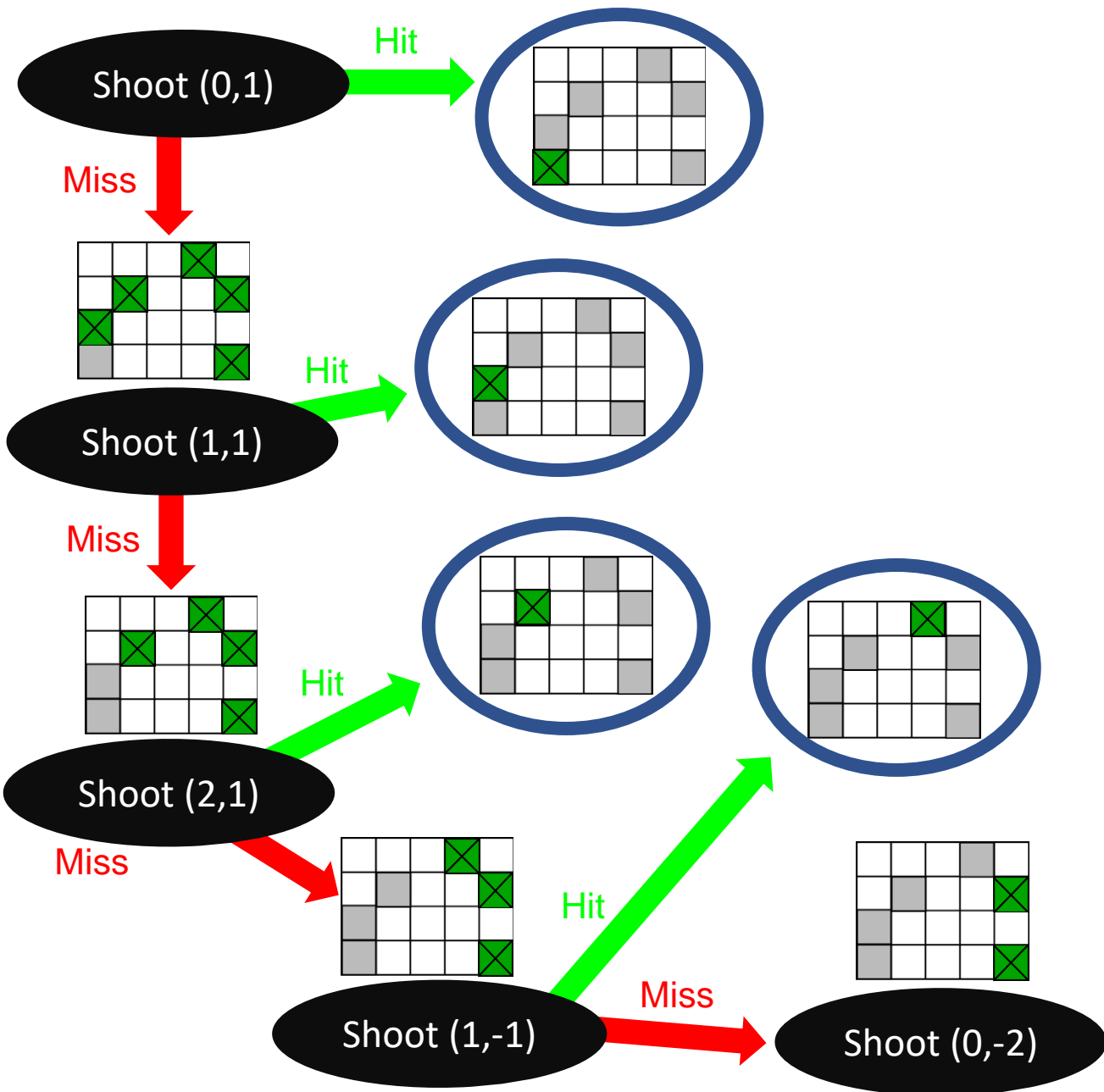


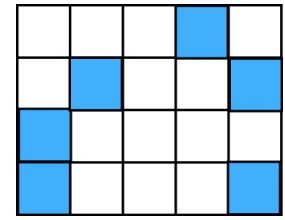
Shape



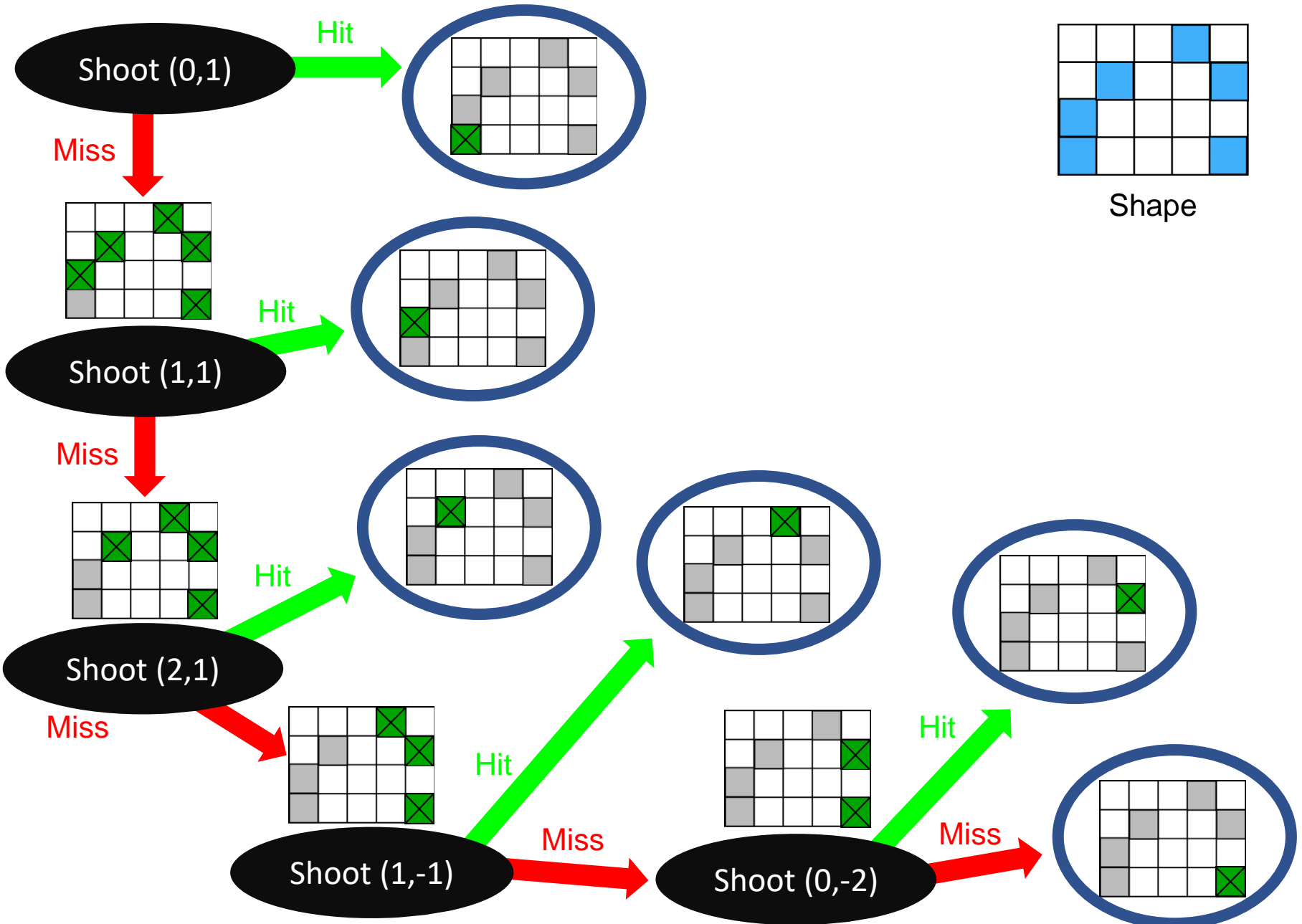


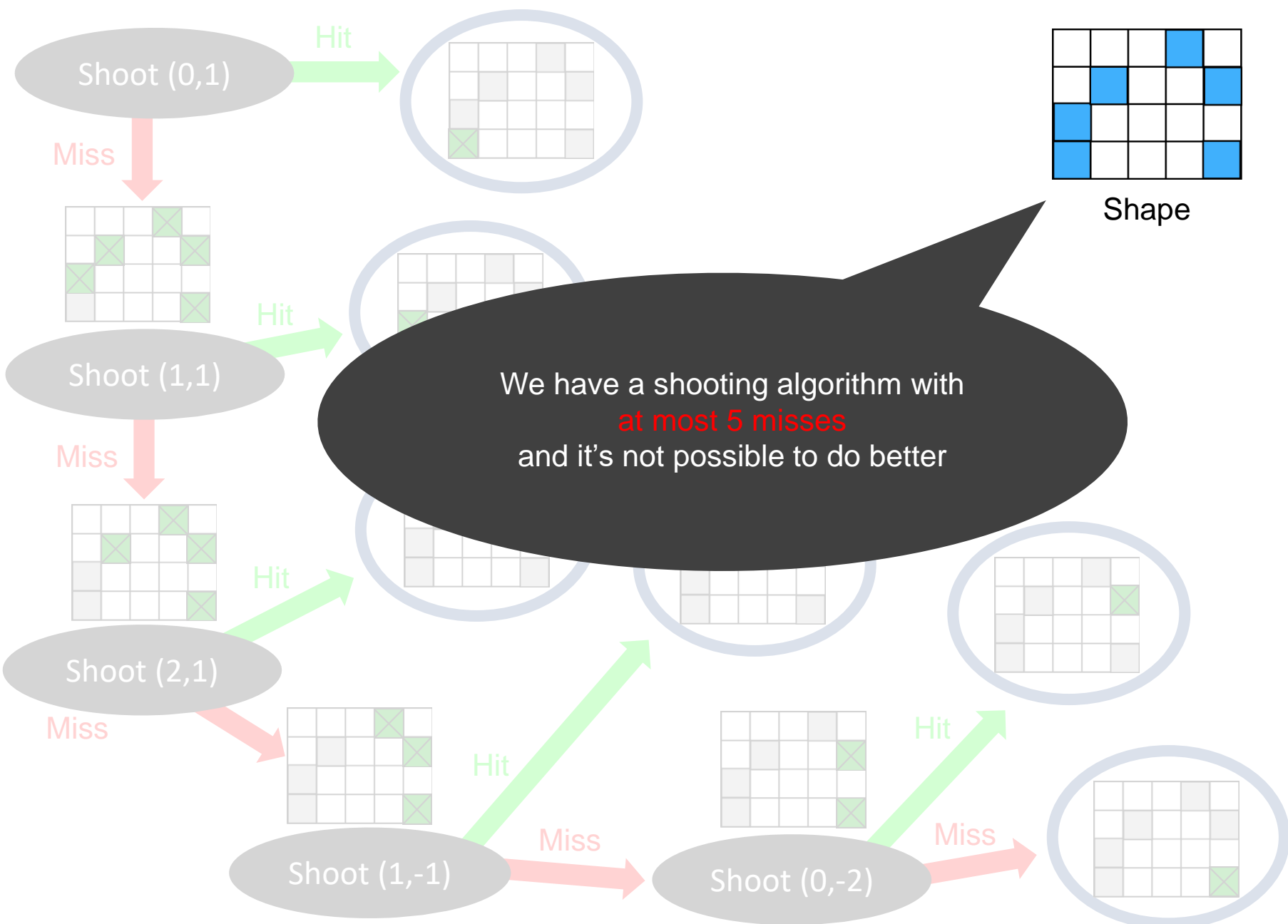
Shape

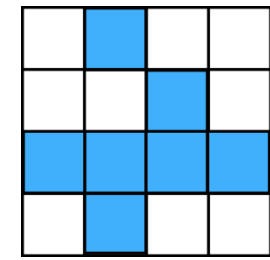




Shape



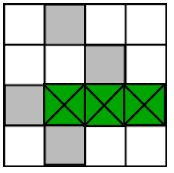




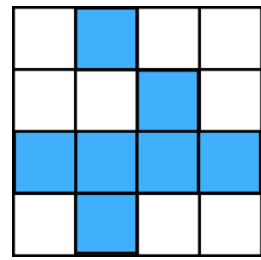
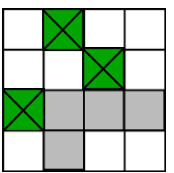
Shape

Shoot (1,-1)

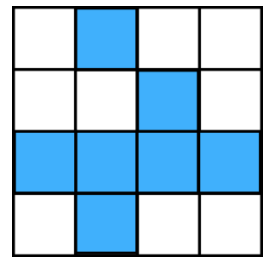
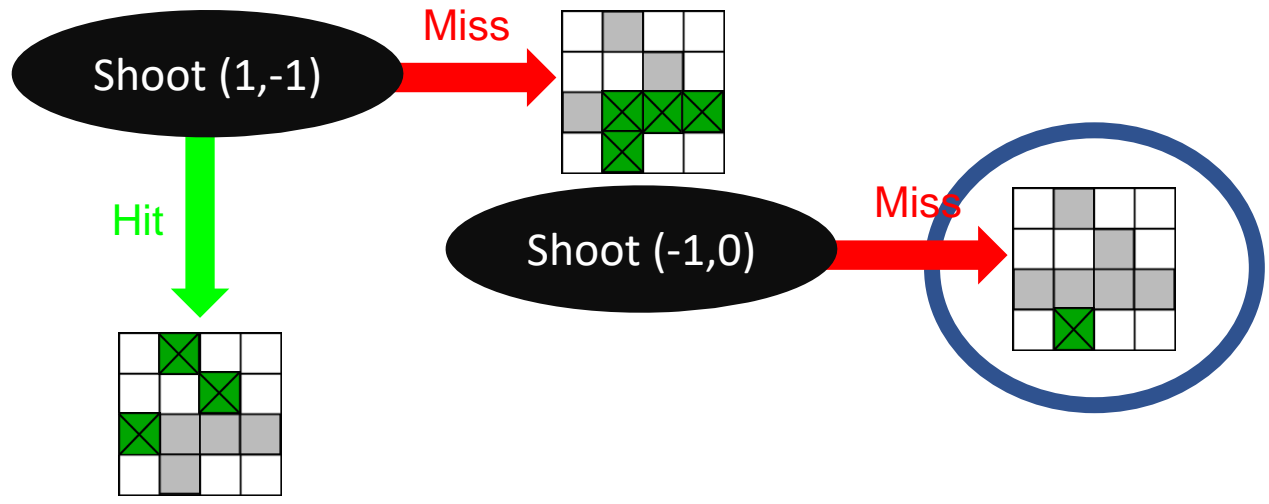
Miss



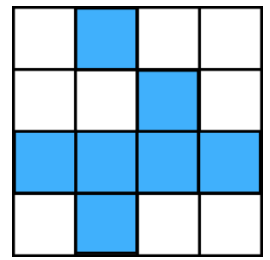
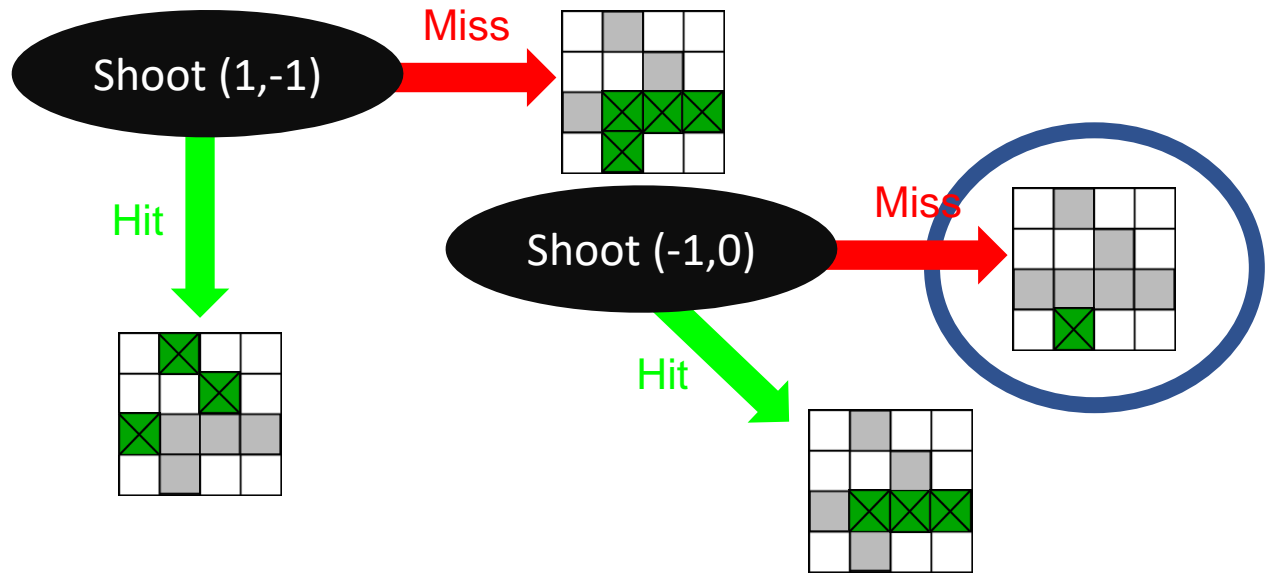
Hit



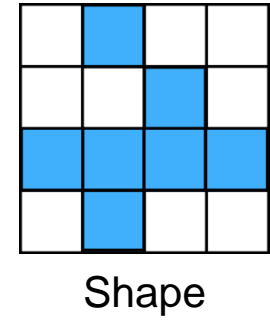
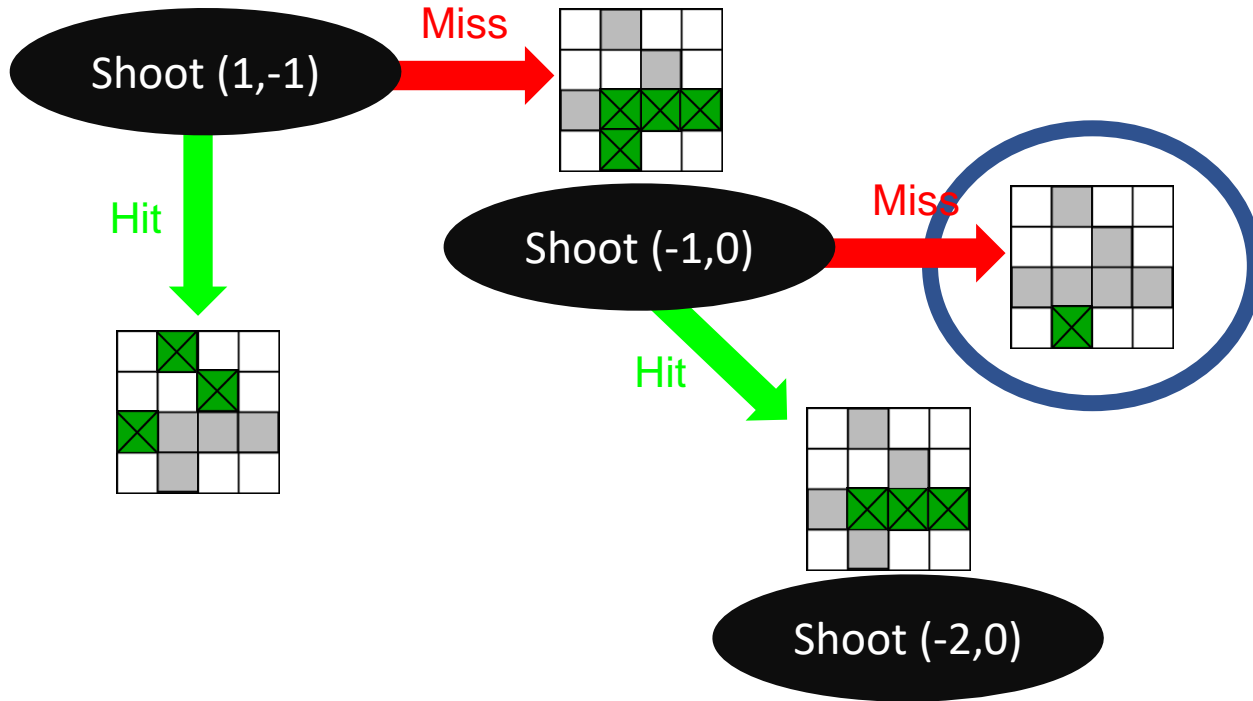
Shape

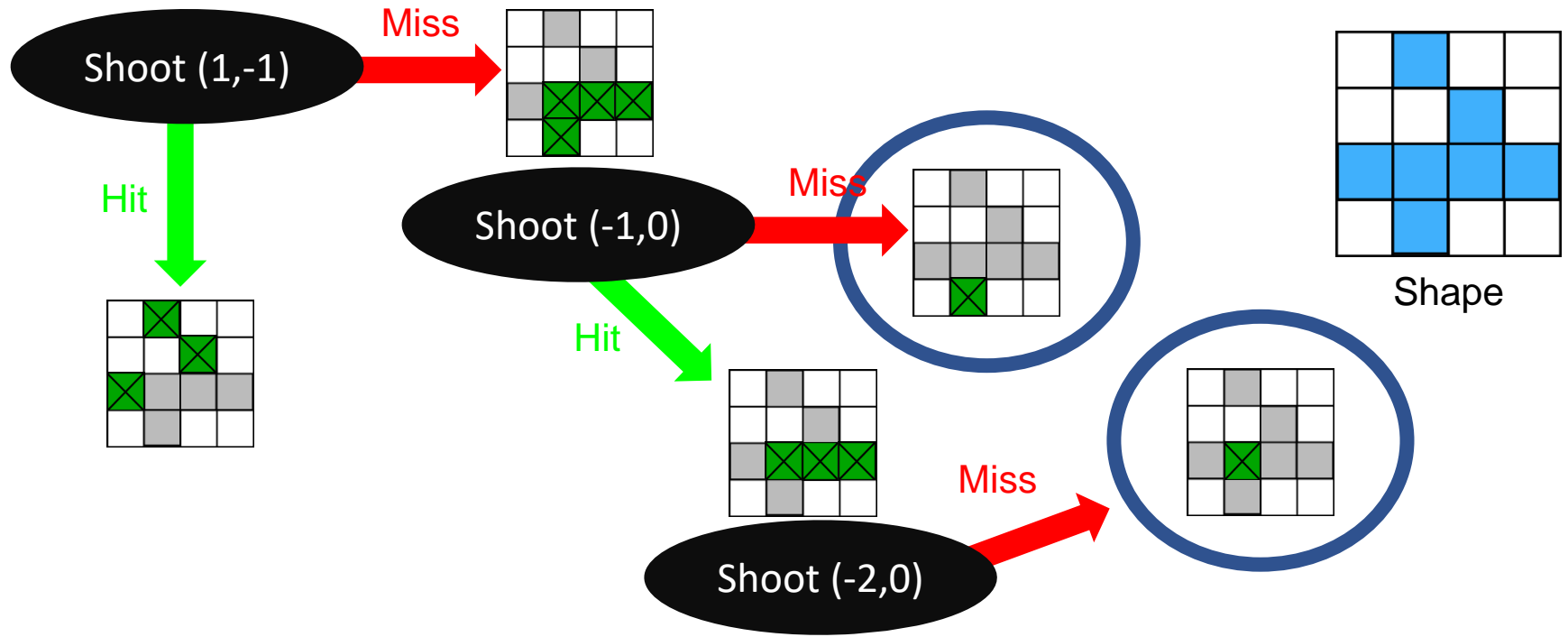


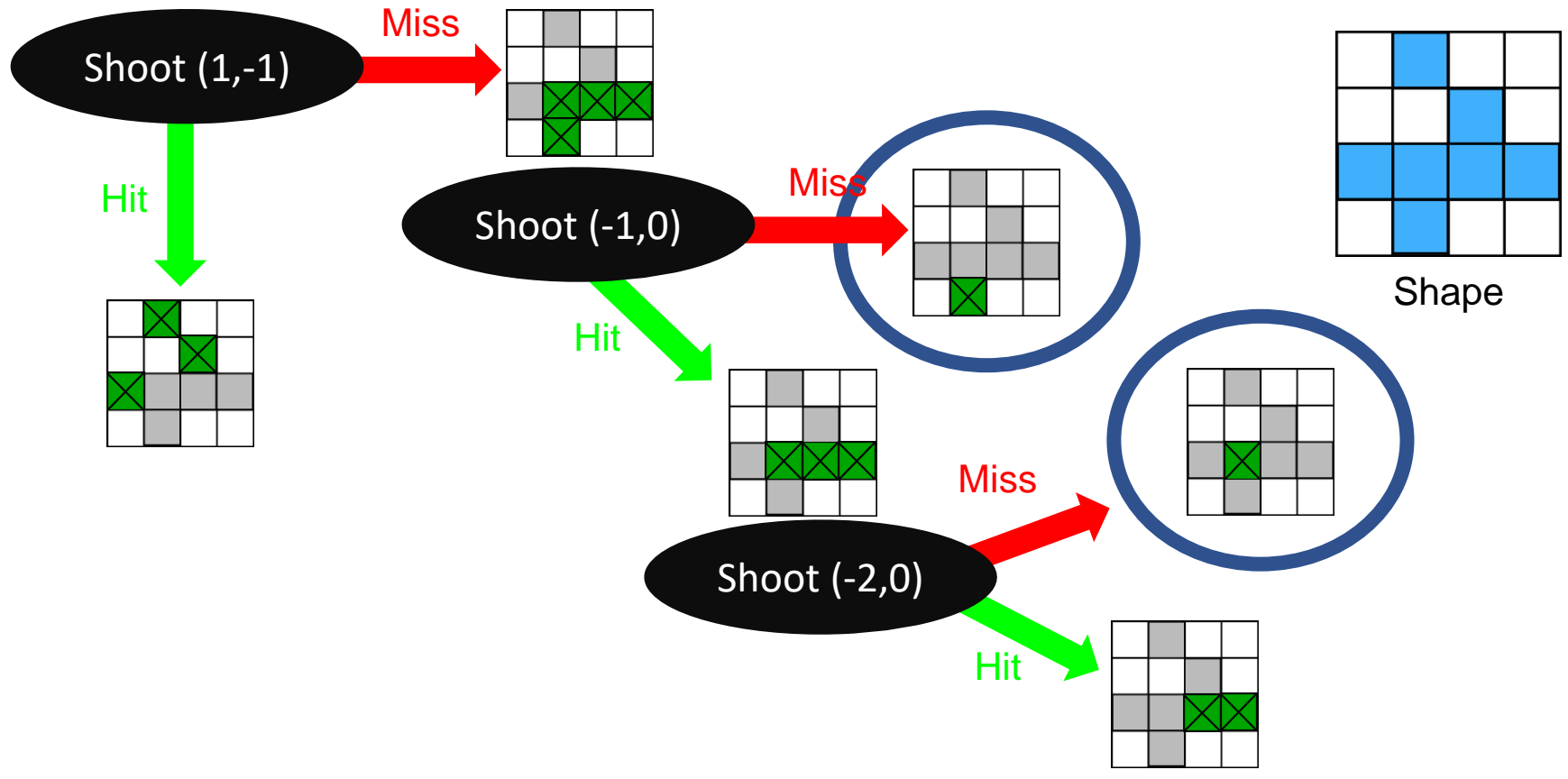
Shape

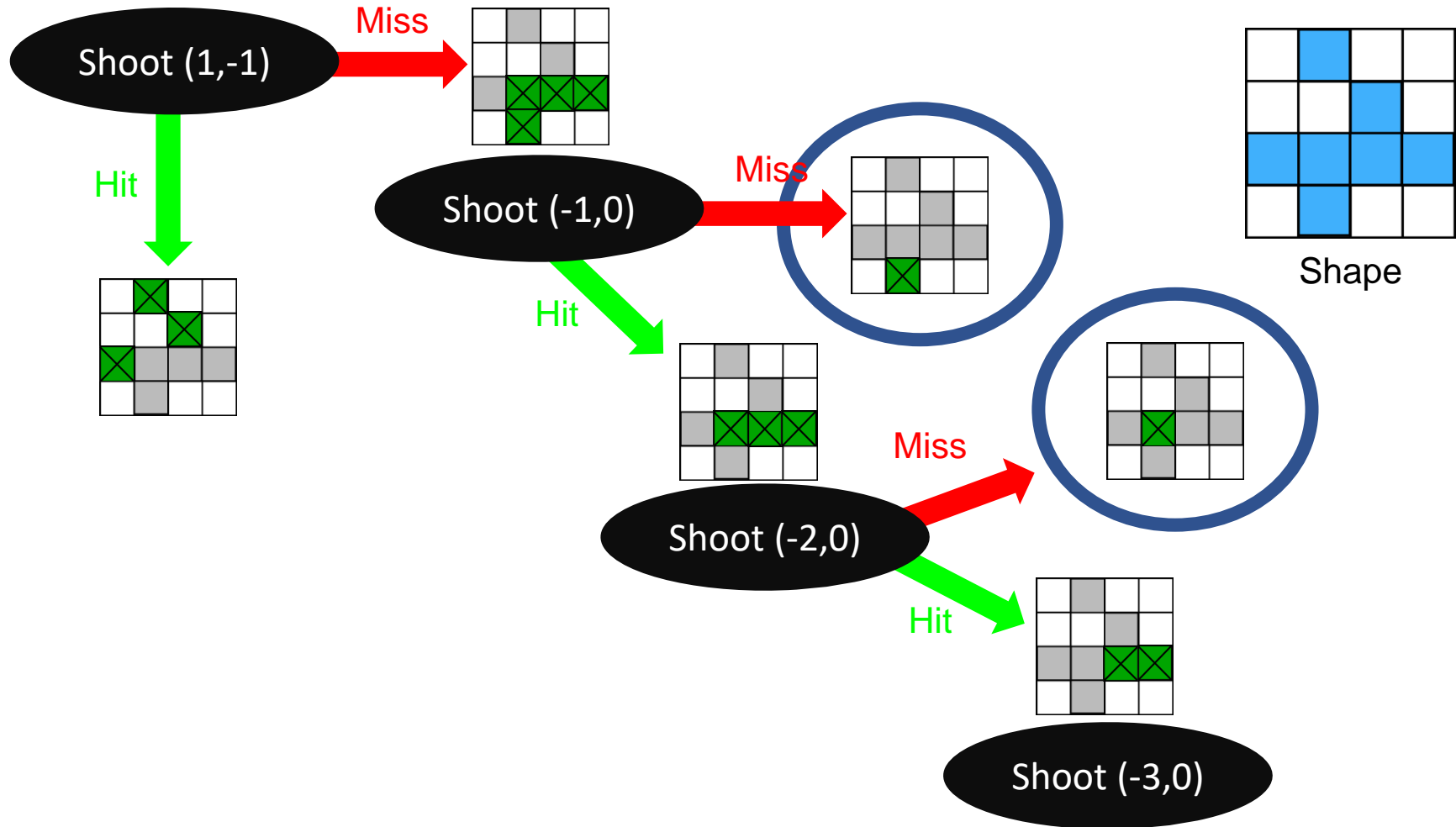


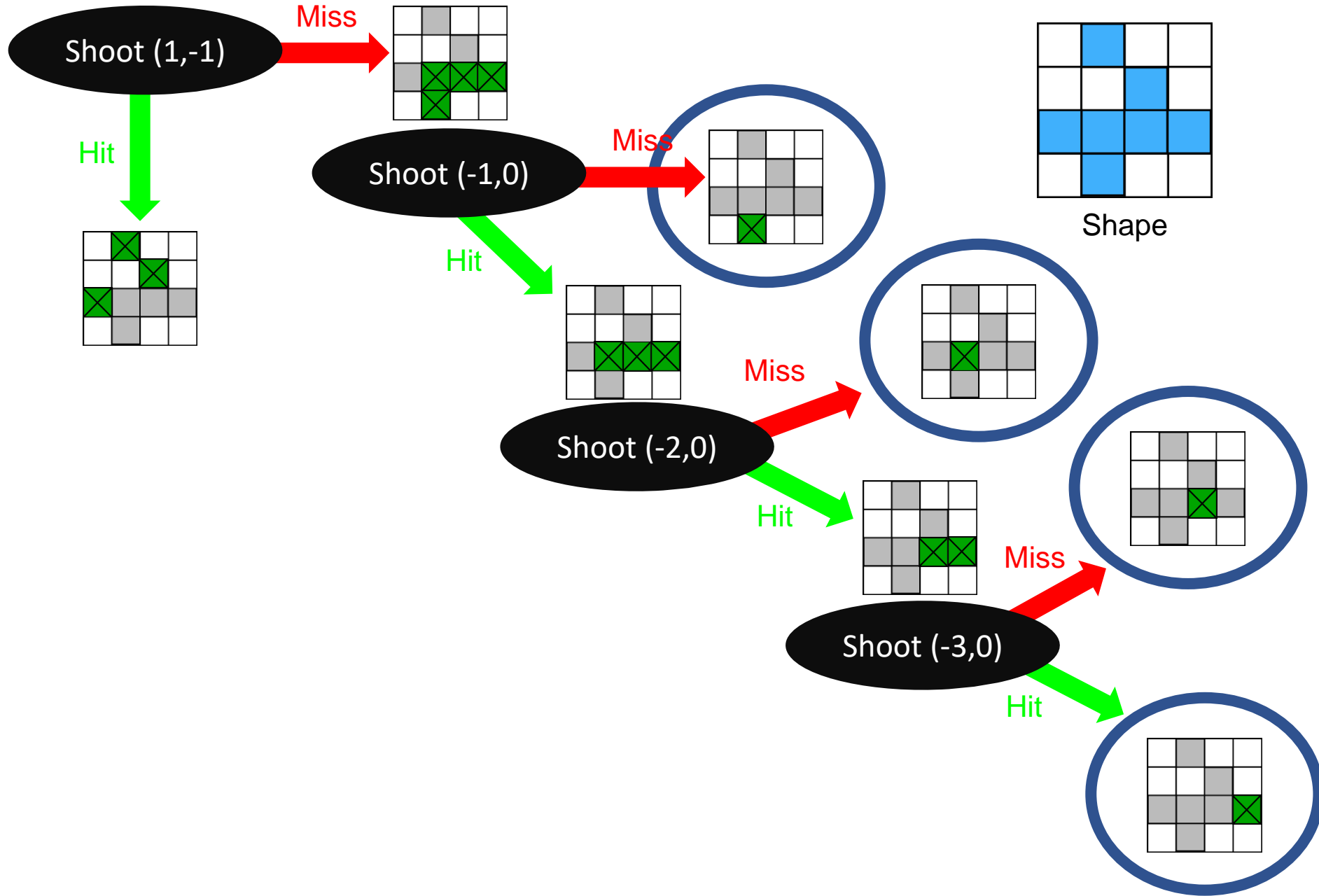
Shape

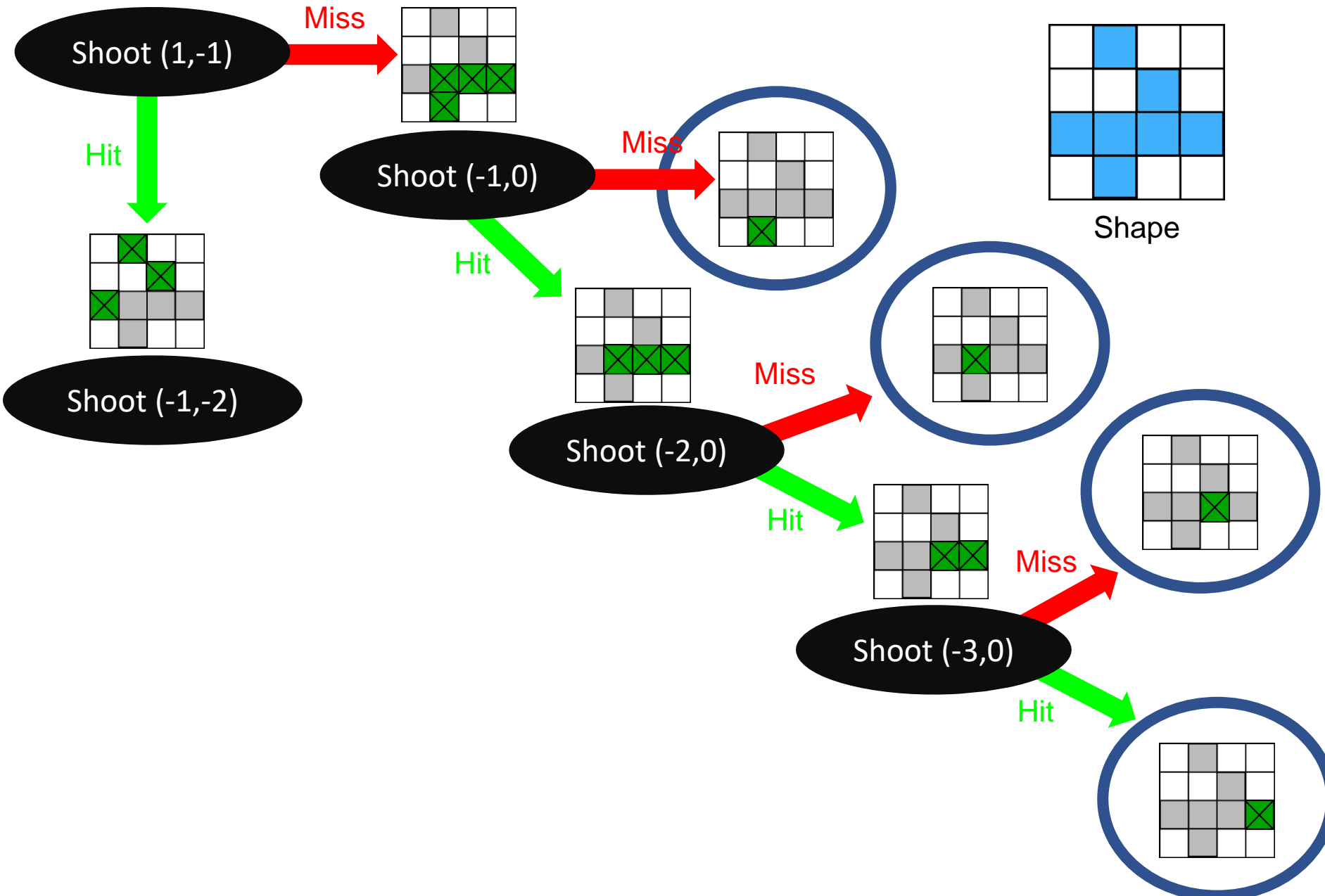


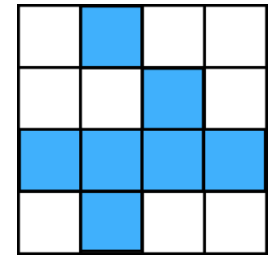






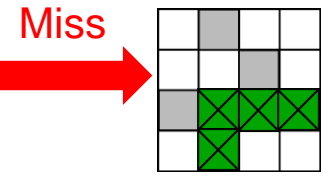






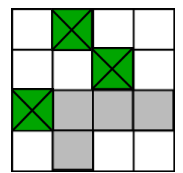
Shape

Shoot (1,-1)

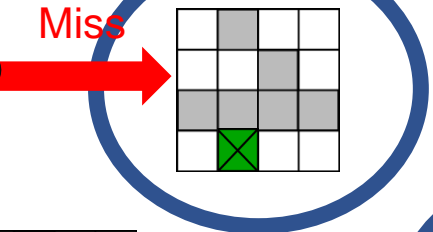


Miss

Hit

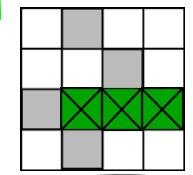


Shoot (-1,0)



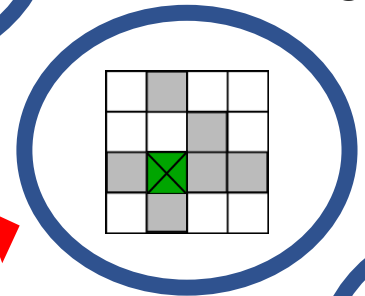
Miss

Hit



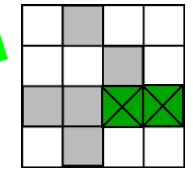
Shoot (-2,0)

Miss



Miss

Hit

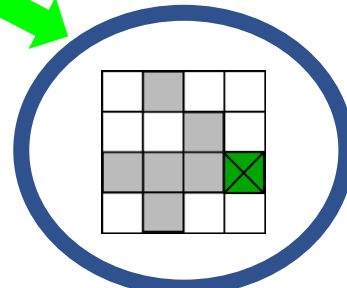


Miss

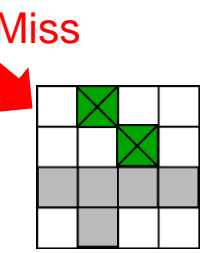
Shoot (-3,0)



Hit

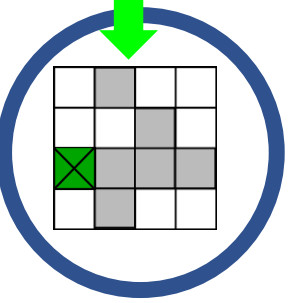


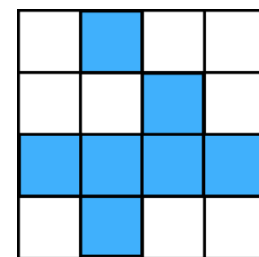
Shoot (-1,-2)



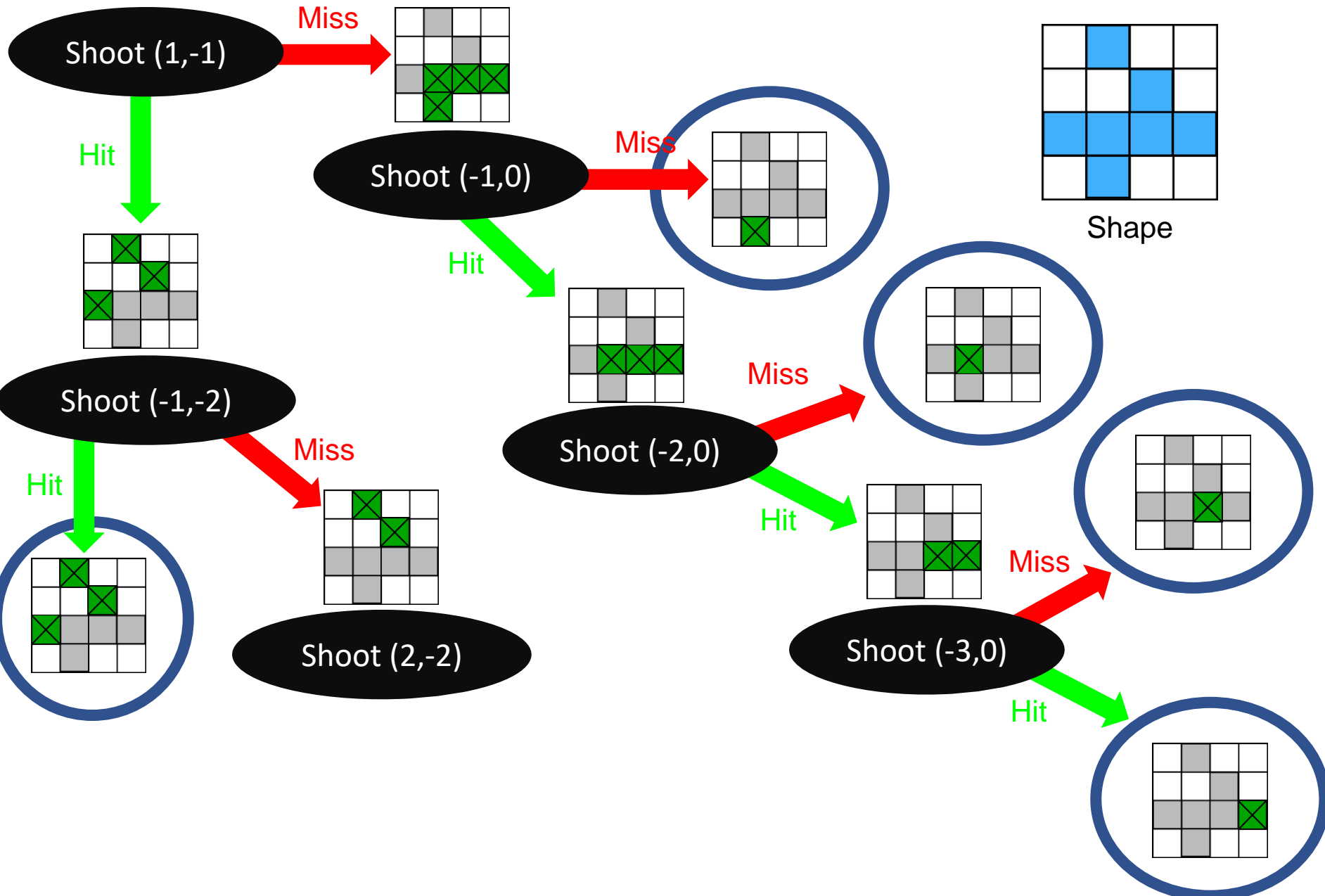
Miss

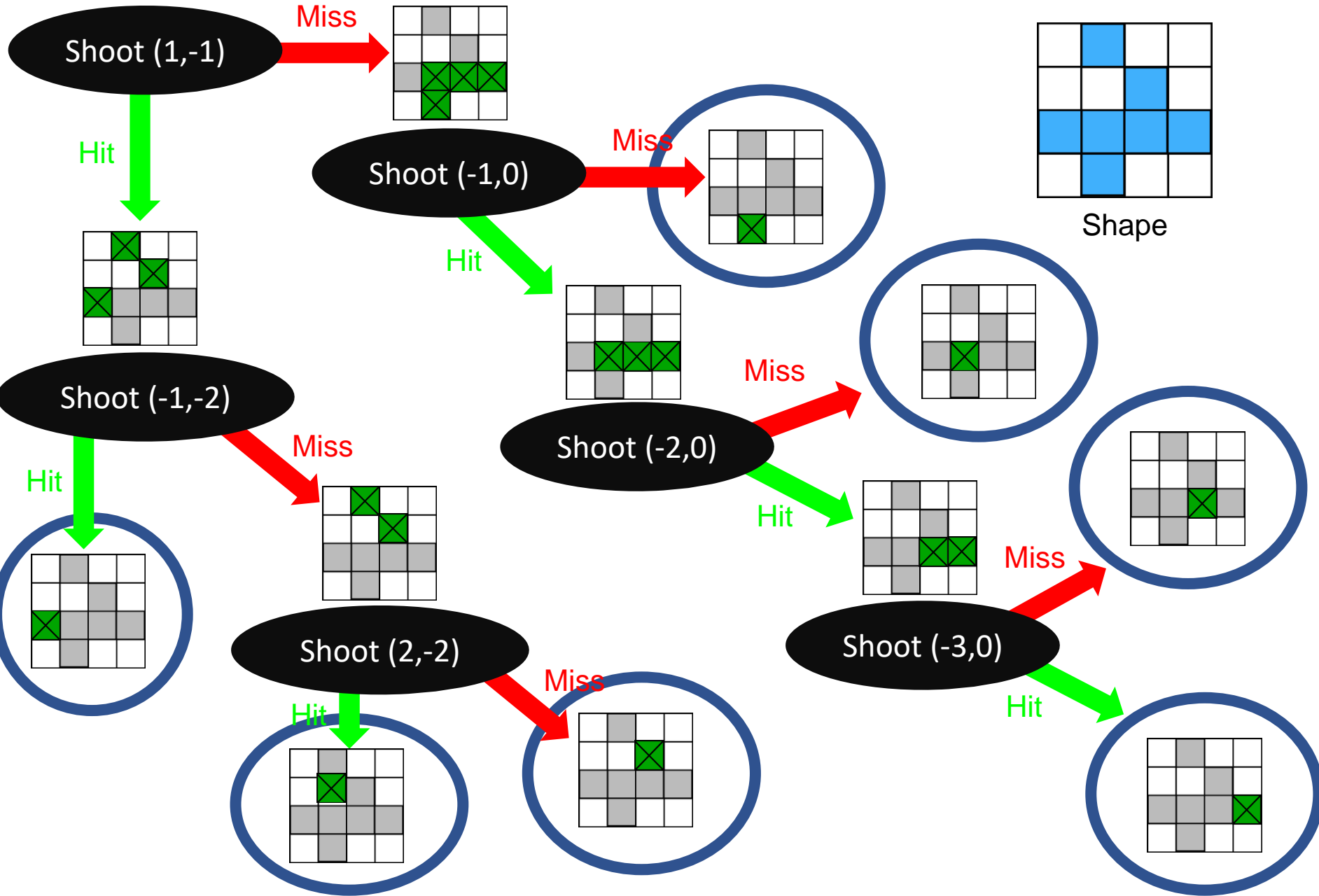
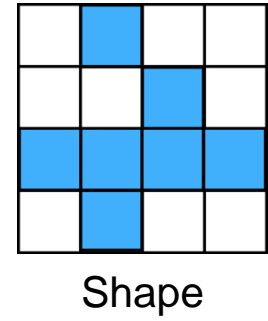
Hit

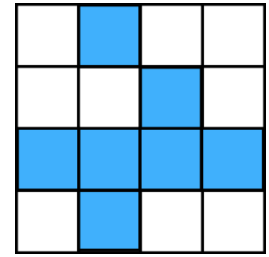




Shape

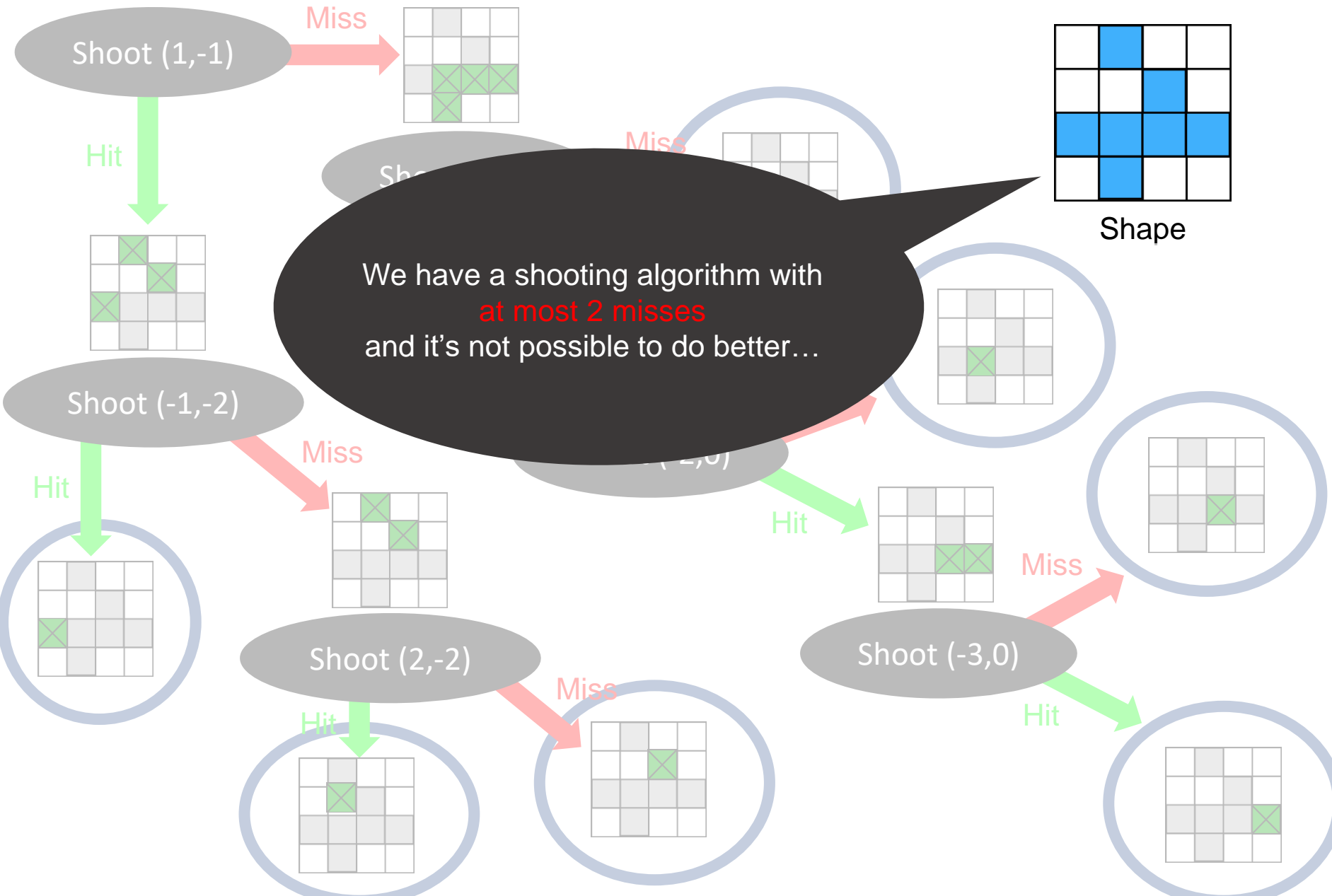






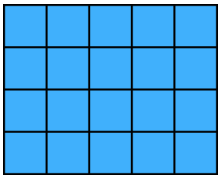
Shape

We have a shooting algorithm with **at most 2 misses** and it's not possible to do better...

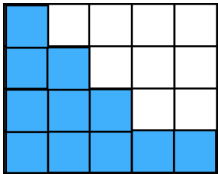




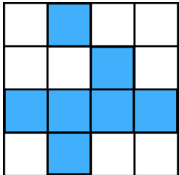
We have an algorithm with at most **1 miss !**



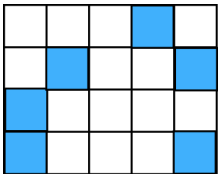
We have an algorithm with at most **2 misses !**



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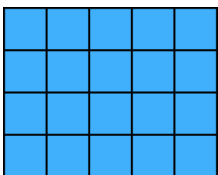
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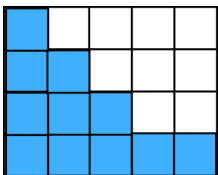
We have an algorithm with at most **5 misses !**



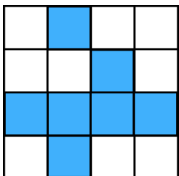
We have an algorithm with at most 1 miss !



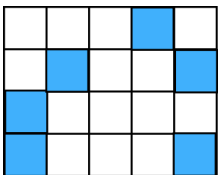
We have an algorithm with at most 2 misses !



We have an algorithm with at most 2 misses !



We have an algorithm with at most 2 misses !

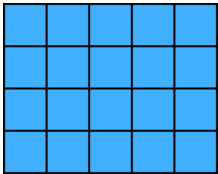


We have an algorithm with at most 5 misses !

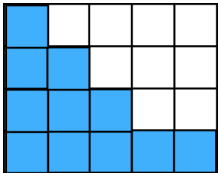
And we cannot do better...



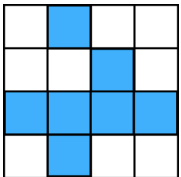
We have an algorithm with at most 1 miss !



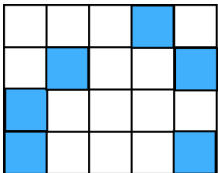
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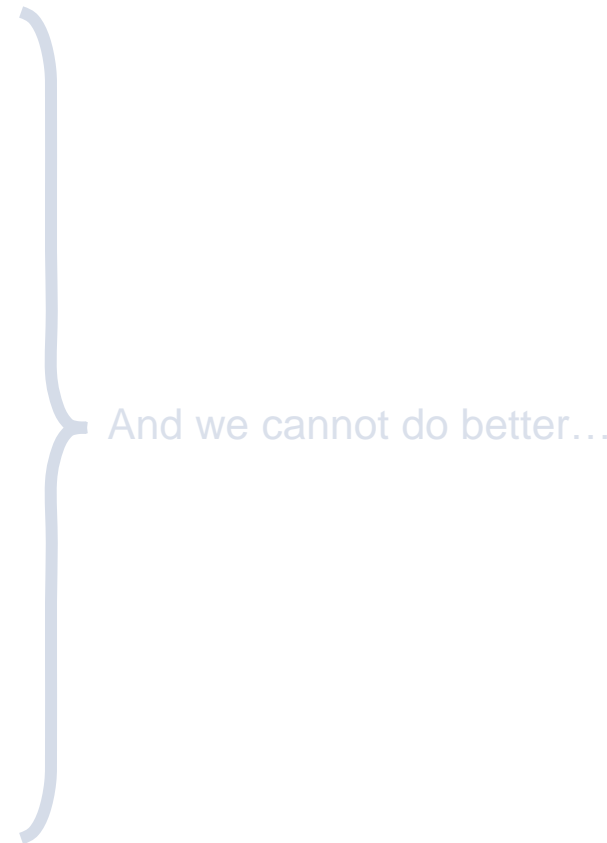
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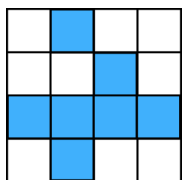
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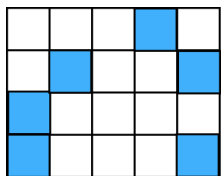
We have an algorithm with at most 5 misses !



This integer is the battleship complexity of each shape



We have an algorithm with at most **2 misses !**



We have an algorithm with at most **5 misses !**

} And we cannot do better...

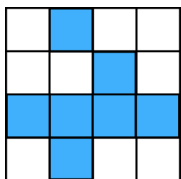


This integer is the *battleship complexity* of each shape

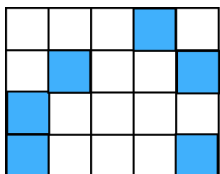
Definition

The *battleship complexity* of a shape is a **minimum** of a **maximum**:
 it is the **minimum** among all **shooting algorithms**
 of the **maximum** number of misses that we can get with this algorithm
 among **all positions** of the ship.

$$\text{Complexity (S)} = \text{Min}_{\text{shooting algorithm}} (\text{Max}_{\text{ship positions}} (\text{number of misses}))$$



We have an algorithm with at most **2 misses !**



We have an algorithm with at most **5 misses !**

} And we cannot do better...



This integer is the *battleship complexity* of each shape

Plan

I

Problem Statement

II

Shooting Algorithms

III

Results

IV

Proofs

Plan

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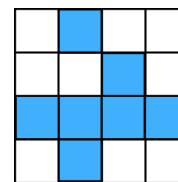
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Complexity(S) = **Min**_{shooting algorithm} (**Max**_{ship positions} (number of misses))

Is it easy to compute ?



Complexity= 2

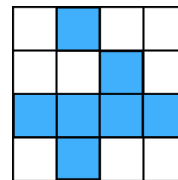
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Is it easy to compute ?

NO!!!! (max for all shooting algorithms!)



Complexity= 2

Definition

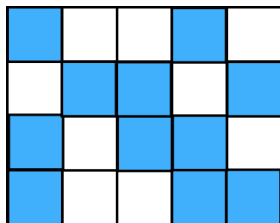
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Complexity(S) = $\text{Min}_{\text{shooting algorithm}} (\text{Max}_{\text{ship positions}} (\text{number of misses}))$

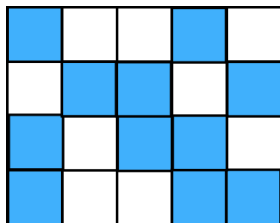
Is it easy to compute ?

NO!!!! (max for all shooting algorithms!)

Any **shooting algorithm**
provides an **upper bound** of
the Battleship Complexity...

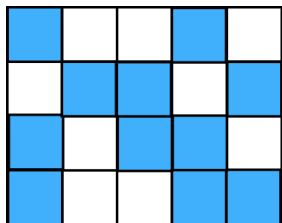


A shape S with n squares



A shape S with n squares

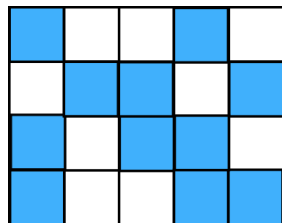
Bound 1
For any shape
 $\text{Complexity}(S) \leq n-1$



A shape S with n squares

Bound 1
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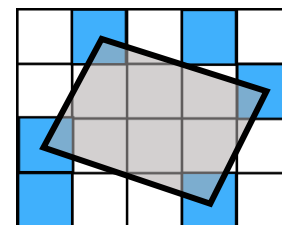
For parallelogram-free
shapes, we have
 $\text{Complexity}(S)=n-1$



A shape S with n squares

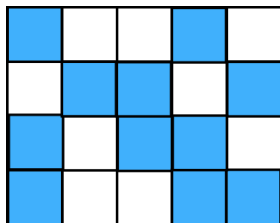
Bound 1
For any shape
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For parallelogram-free
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Non Parallelogram-free shape

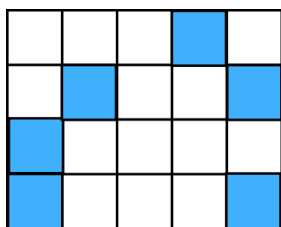
There exists 4 distinct points u, v, s, t in S with
 $v-u=t-s$



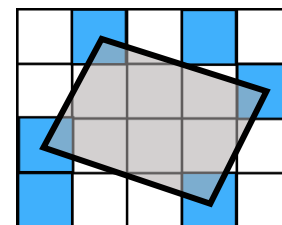
A shape S with n squares

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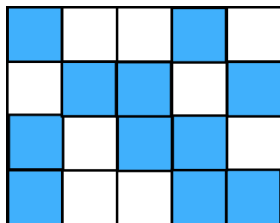


Parallelogram-free shape



Non Parallelogram-free shape

There exists 4 distinct points u, v, s, t in S with
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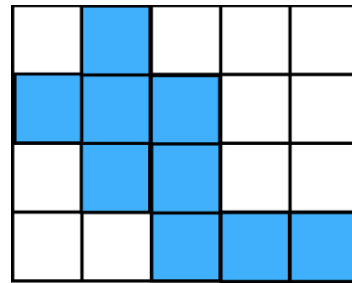


A shape S with n squares

Bound 1
For any shape
 $\text{Complexity}(S) \leq n-1$

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Bound 2
For HV-convex polyominoes
 $\text{complexity}(S) = O(\log(n))$



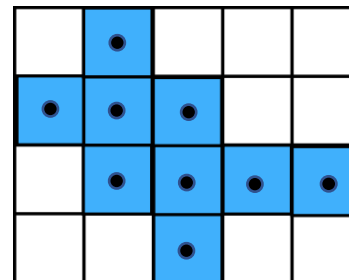
A HV-convex polyomino

It is a 4-connected,
horizontally and vertically convex shape

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Bound 3
For digital convex polyominoes
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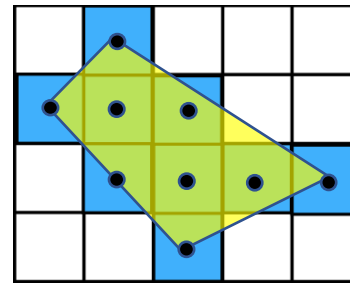


A digital convex shape

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A digital convex shape

Plan

I

Problem Statement

II

Shooting Algorithms

III

Results

IV

Proofs

Plan

I

Problem Statement

II

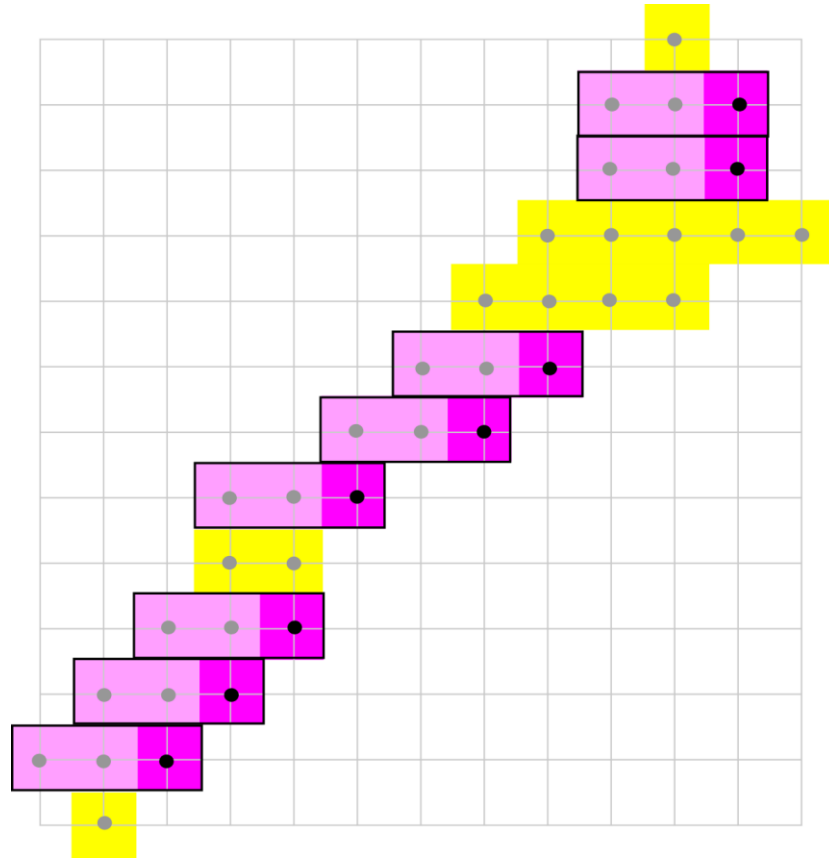
Shooting Algorithms

III

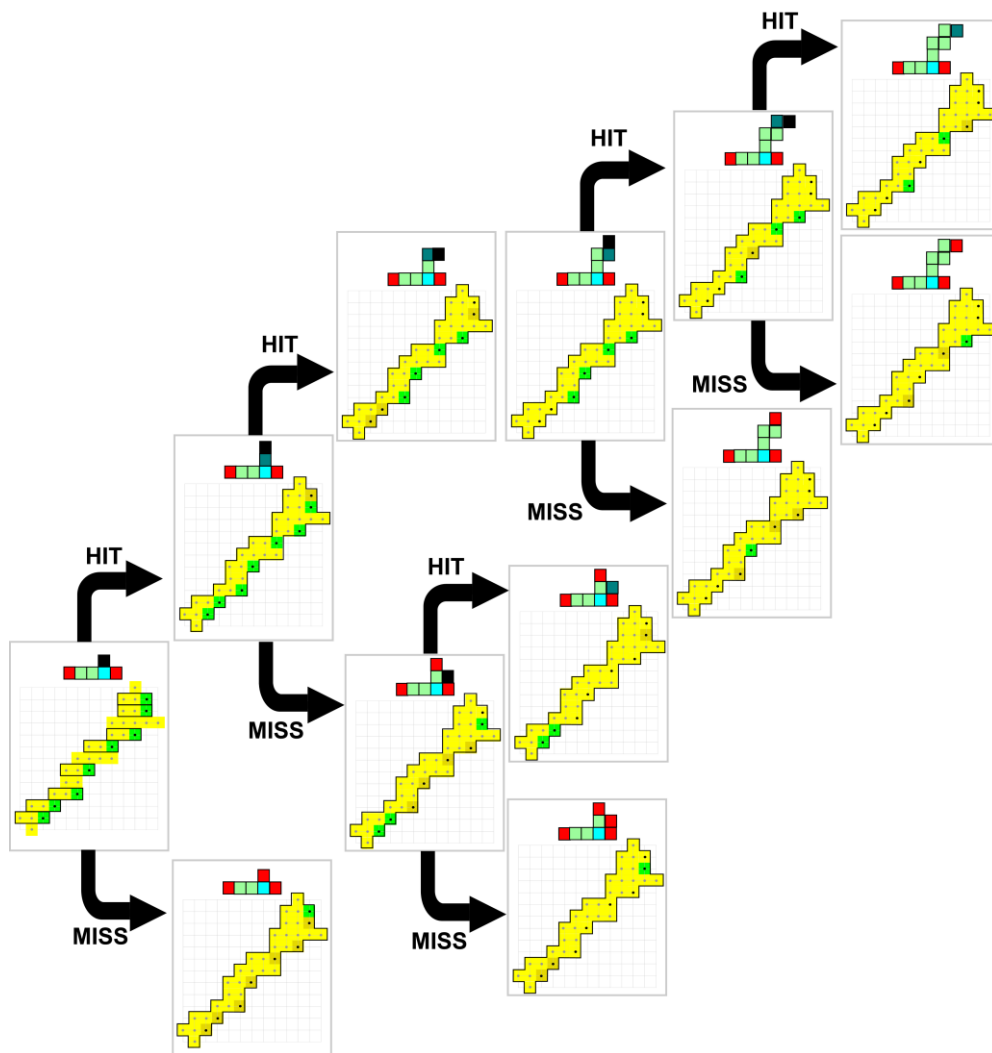
Results

IV

Proofs



We use a property of **monotonicity** of HV-convex polyominoes...



We build a staircase shooting algorithm with at most $O(\log(n))$ misses

Browser tabs: Zimbra: Réception (3632) x Quand l'ivermectine attire l... x Dashboard x Cours : Mathématiques DUT x Efficient Algorithms for Batt... x W Blaschke–Lebesgue theorem x

Address bar: https://en.wikipedia.org/wiki/Blaschke–Lebesgue_theorem

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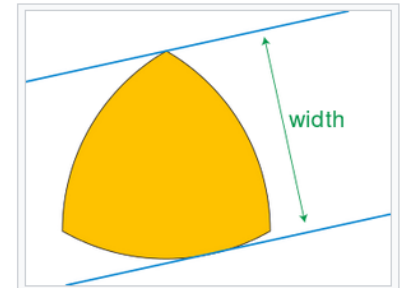
Blaschke–Lebesgue theorem

From Wikipedia, the free encyclopedia

In **plane geometry** the **Blaschke–Lebesgue theorem** states that the **Reuleaux triangle** has the least area of all **curves of given constant width**.^[1] In the form that every curve of a given width has area at least as large as the Reuleaux triangle, it is also known as the **Blaschke–Lebesgue inequality**.^[2] It is named after **Wilhelm Blaschke** and **Henri Lebesgue**, who published it separately in the early 20th century.

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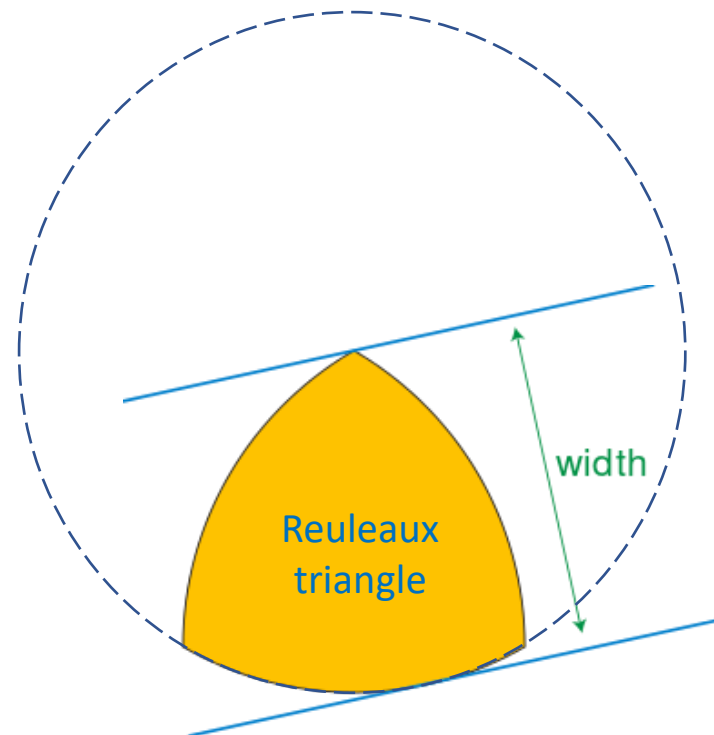
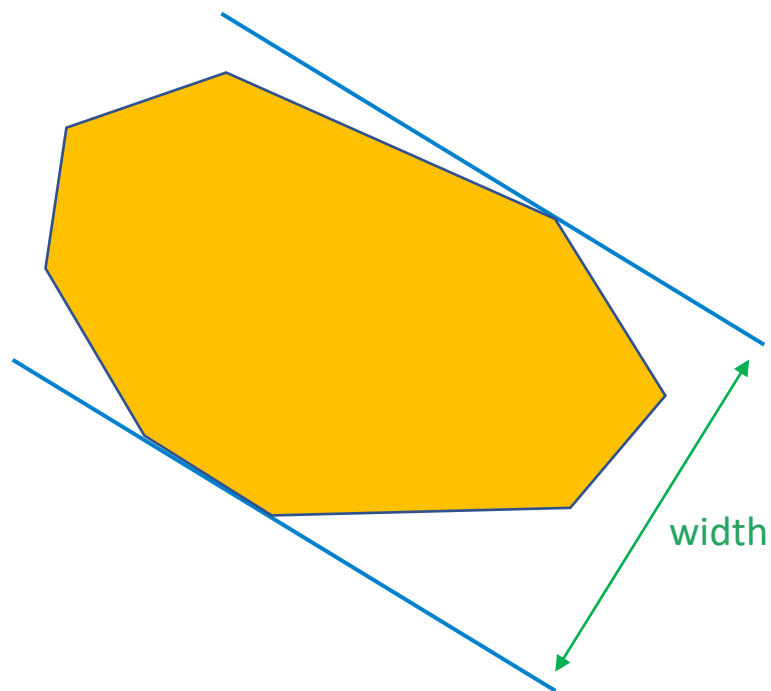
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- [2 History](#)
- [3 In other planes](#)
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A Reuleaux triangle, a curve of constant width whose area is minimum among all convex sets with the same width

Statement [\[edit\]](#)

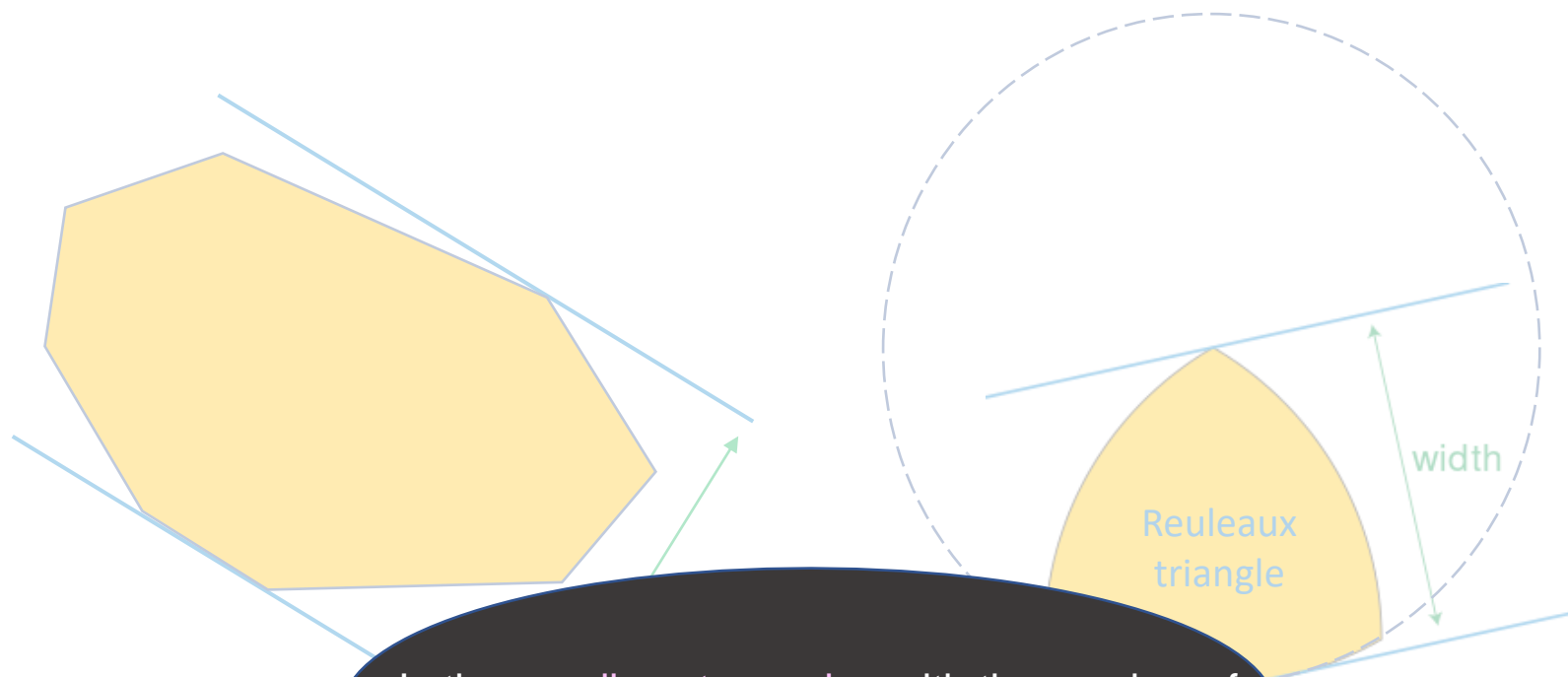
The width of a convex set K in the Euclidean plane is defined as the minimum distance between any two parallel lines that enclose it. The two minimum-distance lines are both necessarily **tangent lines** to K , on opposite sides. A **curve of constant width** is the boundary of a convex set with the property that, for every direction of parallel lines, the two tangent lines with that direction that are tangent to opposite sides of the curve are at a distance equal to the width. These curves include both the circle and the **Reuleaux triangle**, a curved triangle formed from arcs of three equal-radius circles each centered at a crossing point of the other two circles. The area enclosed by a Reuleaux triangle with width w is



Blaschke-Lebesgue Inequality (1914)

The **area** of a convex shape is bounded by
(Equality is achieved for Reuleaux triangles).

$$area \geq \frac{1}{2}(\pi - \sqrt{3})width^2$$



Is there a **discrete version** with the number of points instead of the area ?

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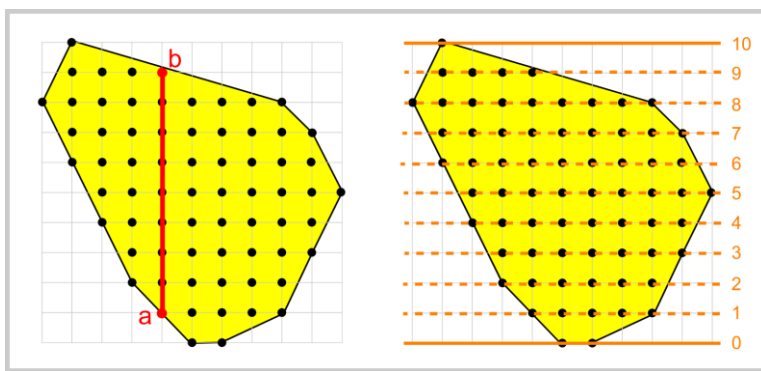
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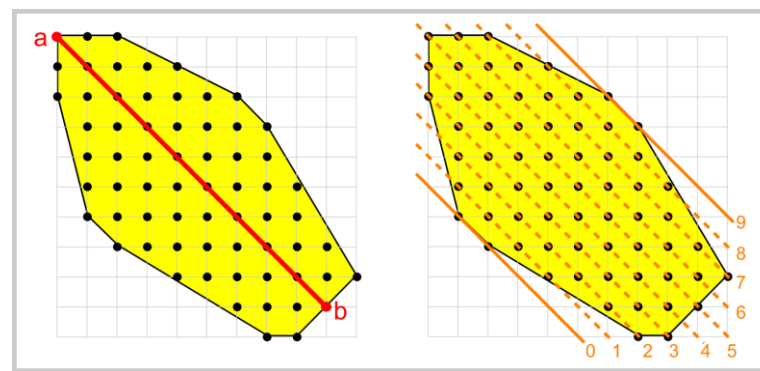
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$diam = 9$

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Pick formula & ...

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Node

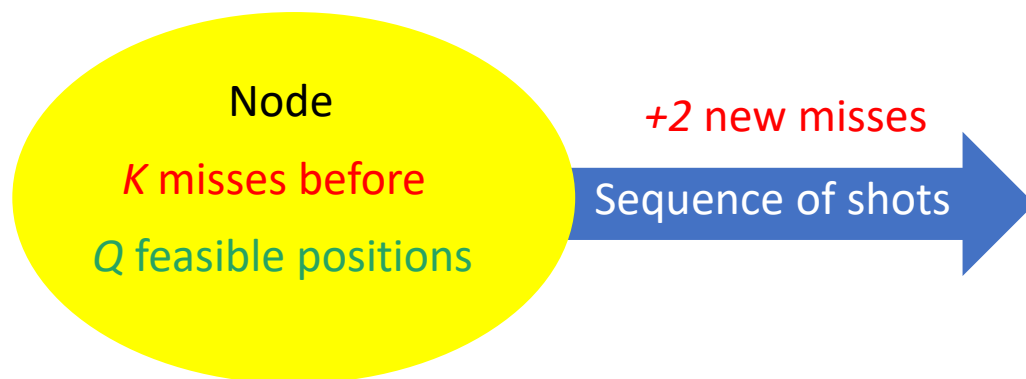
K misses before

Q feasible positions

Discrete Blaschke-Lebesgue Inequality (L. Crombez, G. Da Fonseca, Y.G)

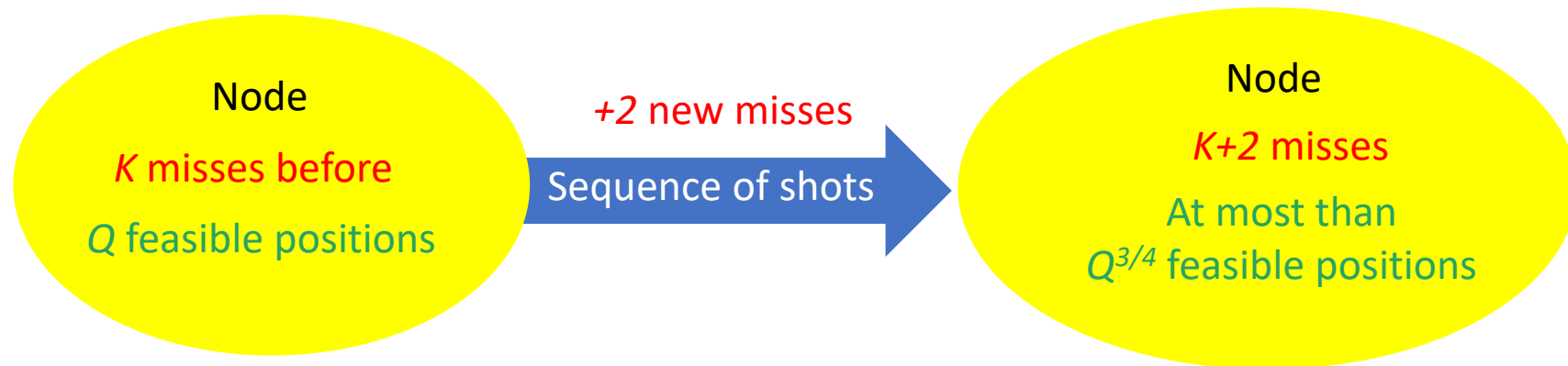
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Conclusion

Three bounds on the Battleship complexity of a shape....

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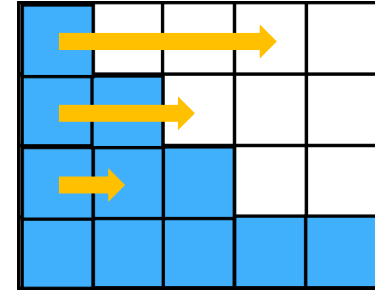
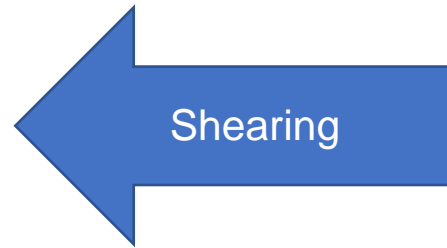
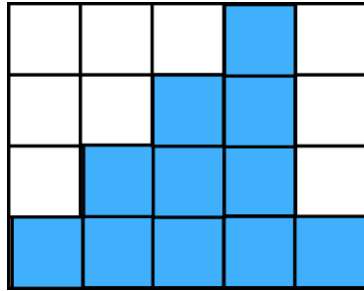
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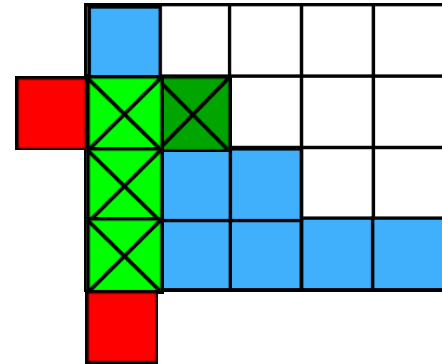
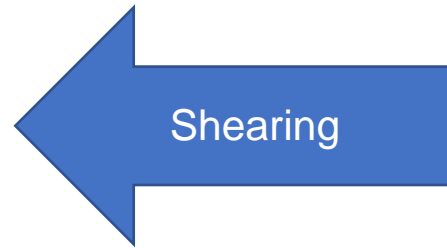
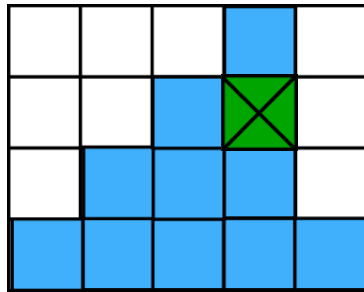
$\text{Complexity}(A+B) \leq \text{Complexity}(A) + \text{Complexity}(B)$?

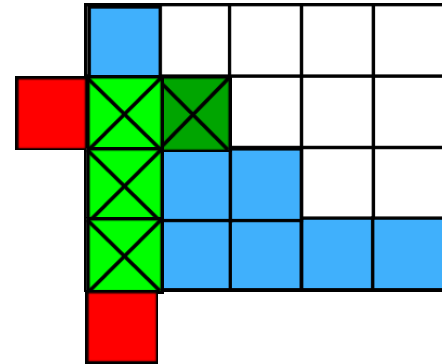
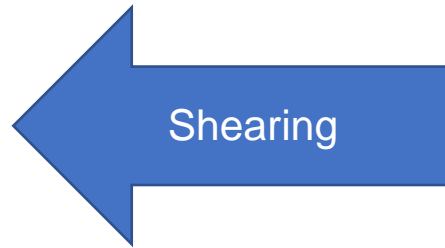
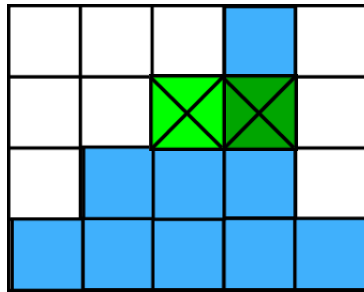
THE END

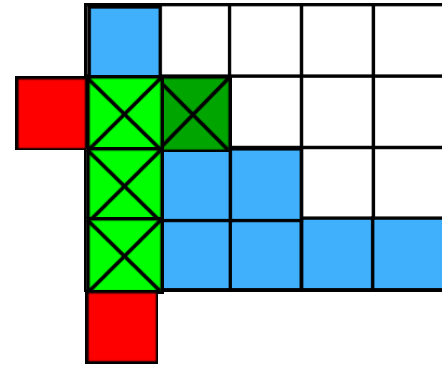
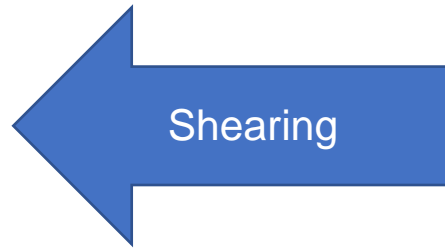
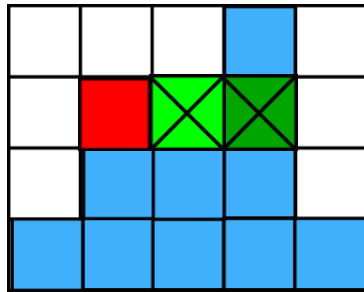
Thanks a lot for your attention...

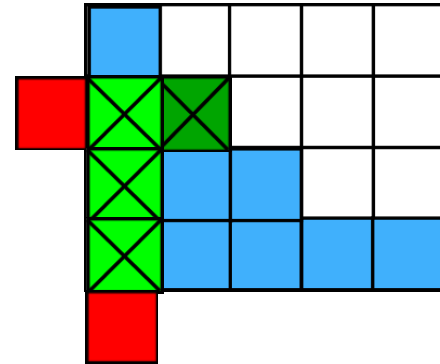
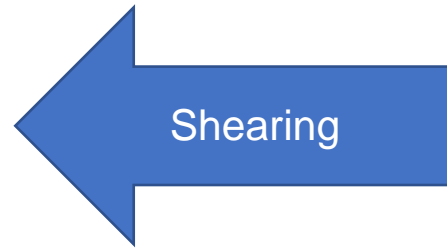
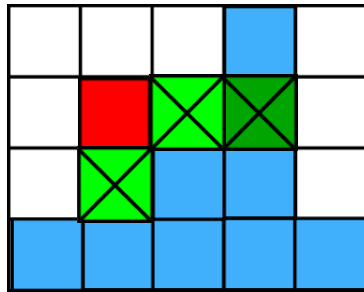


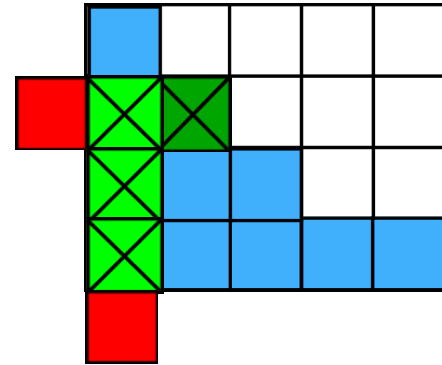
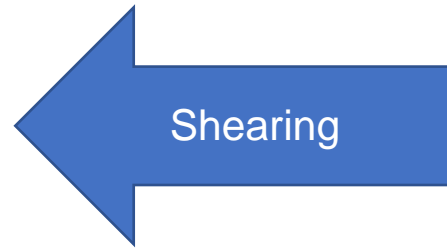
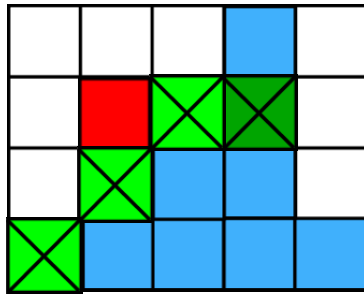
Invariance by isomorphisms

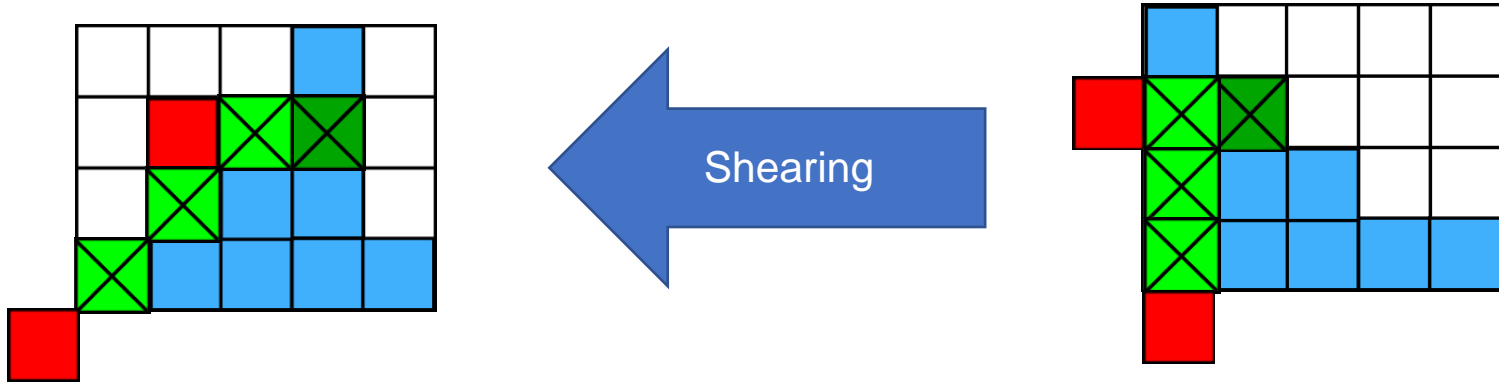












The Battleship complexity is invariant by isomorphisms...